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Research Article

Paranormed Motzkin sequence spaces

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Abstract: In this article, it is obtained two new paranormed sequence spaces $c_0(\mathcal{M},\mathfrak{p})$ and $c(\mathcal{M},\mathfrak{p})$ by the aid of the conservative Motzkin matrix operator $\mathcal{M}$ and is examined some topological properties of these spaces. Also, Schauder basis and the α-, β- and γ-duals are determined. Finally, some new matrix mappings are characterized related new paranormed sequence spaces.

Keywords: Motzkin numbers, Paranormed sequence spaces, Duals, Matrix mappings.

MSC 2020: 40C05, 46A45, 46B45, 47B37.

1. Introduction and preliminaries

Each subspace Γ of the space ω of all sequences with real terms is named as a sequence space. The spaces of bounded, convergent, null and q-summable sequences with real terms are known by the abbreviations ℓ_∞, c, c_0 and ℓ_q, respectively. The aforementioned spaces are Banach spaces due to the norms $\|u\|_{\ell_\infty} = \|u\|_c = \|u\|_{c_0} = \sup_{s\in\mathbb{N}}|u_s|$ and $\|u\|_{\ell_q} = (\sum_s |u_s|^q)^{1/q}$ for $1 \le q < \infty$, where the notion $\sum_s$ purports the summation $\sum_{s=0}^{\infty}$ and $\mathbb{N} = \{0,1,2,3,\dots\}$.. The acronym $e^{(s)}$ denotes a sequence whose s^{th} term is 1 and the rest of the terms are zero and $e = (1,1,1,\dots)$.

Consider the space $\Gamma \subset \omega$ and the operator $\wp$ from Γ to real number' set $\mathbb{R}$. In that case, the pair $(\Gamma,\wp)$ is called as a paranormed sequence space if the followings are hold for each $u,v \in \Gamma$ and $\lambda \in \mathbb{R}$.

(p1) $\wp(u) \geq 0$ and the fact $u = \theta$ implies that $\wp(u) = 0$ for the zero element θ of Γ.

(p2) $\wp(-u) = \wp(u)$.

(p3) $\wp(u + \nu) \leq \wp(u) + \wp(\nu)$ (subadditive).

(p4) $|\lambda_s - \lambda| \to 0$ and $\wp(u_s - u) \to 0$ implies that $\wp(\lambda_s u_s - \lambda u) \to 0$ for $(\lambda_s) \in \omega$ and $(u_s) \in \Gamma$ as $s \to \infty$.

In that case, the operator $\wp$ is named as a paranorm on Γ.

Let us suppose that the bounded sequences $(\mathfrak{p}_r)$ and $(\mathfrak{q}_r)$ of strictly positive real numbers with $\sup \mathfrak{p}_r = \mathcal{S}$ and $\mathcal{Q} = \max\{1, \mathcal{S}\}$.

The concept of paranormed sequence spaces, that is, paranormed sequence spaces $c_0(\mathfrak{p})$, $c(\mathfrak{p})$, $\ell_\infty(\mathfrak{p})$ and $\ell(\mathfrak{p})$, was developed by Maddox **(1968)** (also see **(Nakano, 1951)** and **(Simons, 1969)**) and these spaces are expressed in the following form:

$$
\begin{aligned}
c_0(\mathfrak{p}) &= \left\{u = (u_s) \in \omega \colon \lim_{s\to\infty} |u_s|^{\mathfrak{p}_s} = 0\right\}, \\
c(\mathfrak{p}) &= \left\{u = (u_s) \in \omega \colon \lim_{s\to\infty} |u_s - \kappa|^{\mathfrak{p}_s} = 0, \exists \kappa \in \mathbb{R}\right\}, \\
\ell_\infty(\mathfrak{p}) &= \left\{u = (u_s) \in \omega \colon \sup_{s\in\mathbb{N}} |u_s|^{\mathfrak{p}_s} < \infty\right\}
\end{aligned}
$$

and

$$
\ell(\mathfrak{p}) = \left\{u = (u_s) \in \omega \colon \sum_{s=0}^{\infty} |u_s|^{\mathfrak{p}_s} < \infty\right\}.
$$

The spaces mentioned above are generally called as Maddox's spaces. The spaces $c_0(\mathfrak{p})$, $c(\mathfrak{p})$, $\ell_\infty(\mathfrak{p})$ and $\ell(\mathfrak{p})$ are complete paranormed sequence spaces with the paranorms

$$
\wp_\infty(u) = \sup_{s\in\mathbb{N}} |u_s|^{\mathfrak{p}_s/\mathcal{Q}} \text{ iff } \inf_{s\in\mathbb{N}} \mathfrak{p}_s > 0 \text{ and } \wp(u) = \left(\sum_{s=0}^{\infty} |u_s|^{\mathfrak{p}_s}\right)^{1/\mathcal{Q}},
$$

respectively.

Given spaces $\Gamma, \Psi \subset \omega$, the set $M(\Gamma * \Psi)$ is defined as follows:

$$
M(\Gamma * \Psi) = \{\mu = (\mu_r) \in \omega \colon \mu u = (\mu_r u_r) \in \Psi,\ \forall u \in \Gamma\}.
$$

Then, the α-, β- and γ-duals of the space Γ are given by:

$$\Gamma^{\alpha} = M(\Gamma * \ell_1),\ \Gamma^{\beta} = M(\Gamma * cs) \text{ and } \Gamma^{\gamma} = M(\Gamma * bs).$$

For an infinite matrix $D = (d_{rs})$ possessing entries from the real field, D_r denotes the r^{th} row. The D-transform of a sequence $u = (u_s) \in \omega$, denoted by $(Du)_r$, is described as $\sum_{s=0}^{\infty} d_{rs}u_s$, assuming that the series converges for each $r \in \mathbb{N}$.

Let Γ and Ψ be sequence spaces. A matrix D is called as a matrix mapping from Γ to Ψ, if for every $u \in \Gamma$, the image Du belongs to Ψ. The class of all such matrices that effectuate a mapping from Γ to Ψ is represented with $(\Gamma : \Psi)$. Additionally, the notation Γ_D is employed to represent the set of all sequences for which the D-transform is contained in Γ, as expressed by:

$$\Gamma_D = \{u \in \omega : Du \in \Gamma\}. \tag{1}$$

2. Motzkin numbers and associated sequence spaces

The first basic informations about the Motzkin number sequence, one of the most interesting number sequences, are obtained from Motzkin's study **(Motzkin, 1948)**. The r^{th} Motzkin number represents the number of different situations in which r distinct points on a circle can be joined by non-intersecting chords in mathematics. To point out a detail here, the chords do not need to touch all points on the circle in each case. The first few terms of the Motzkin number sequence $(M_r)_{r\in\mathbb{N}}$, which has an important place in combinatorics, number theory and geometry, are given as follows:

$$1, 1, 2, 4, 9, 21, 51, 127, 323, 835, 2188, 5798, \ldots.$$

The recurrence relations of M_r are given the by following way:

$$M_r = M_{r-1} + \sum_{s=0}^{r-2} M_s M_{r-s-2} = \frac{2r+1}{r+1} M_{r-1} + \frac{3r-3}{r+2} M_{r-2}.$$

Another relation provided by the Motzkin numbers is given below:

$$M_{r+2} - M_{r+1} = \sum_{s=0}^{r} M_s M_{r-s}, \text{ for } r \geq 0.$$

The generating function $m(u) = \sum_{r=0}^{\infty} M_r u^r$ of the Motzkin numbers satisfies

$$u^2 + [m(u)]^2 + (u-1)m(u) + 1 = 0$$

and is described by

$$m(u) = \frac{1 - u - \sqrt{1 - 2u - 3u^2}}{2u^2}.$$

Expression on Motzkin numbers with the help of integral function is as follows:

$$M_r = \frac{2}{\pi}\int_0^{\pi} \sin^2 u(2cosu + 1)^r du.$$

They also have the asymptotic behaviour

$$M_r \sim \frac{1}{2\sqrt{\pi}}\left(\frac{3}{r}\right)^{\frac{3}{2}} 3^r,\ r \to \infty.$$

Furthermore, it is known that

$$\lim_{n\to\infty} \frac{M_r}{M_{r-1}} = 3.$$

In addition to the basic information stated above, readers can benefit from the studies Aigner **(1998)**, Barrucci et al. **(1991)**, and Donaghey and Shapiro **(1977)** for more comprehensive content about Motzkin numbers and related subjects.

Erdem et al. **(2024)**, constructed the Motzkin matrix $\mathcal{M} = (\mathfrak{m}_{rs})_{r,s\in\mathbb{N}}$ with the help of Motzkin numbers as follows:

$$\mathfrak{m}_{rs} := \begin{cases} \dfrac{M_s M_{r-s}}{M_{r+2} - M_{r+1}} & , \quad \text{if } 0 \le s \le r, \\ 0 & , \quad \text{if } s > r. \end{cases} \tag{2}$$

Furthermore, it is known from the aforementioned study that the Motzkin matrix $\mathcal{M}$ is conservative, that is $\mathcal{M} \in (c : c)$ and it is given the inverse $\mathcal{M}^{-1} = (\mathfrak{m}_{rs}^{-1})$ of the Motzkin matrix $\mathcal{M}$ as

$$\mathfrak{m}_{rs}^{-1} := \begin{cases} (-1)^{r-s} \dfrac{M_{s+2} - M_{s+1}}{M_r} \pi_{r-s} & , \quad \text{if } 0 \le s \le r, \\ 0 & , \quad \text{if } s > r, \end{cases} \tag{3}$$

where $\pi_0 = 0$ and

$$\pi_r = \begin{vmatrix} M_1 & M_0 & 0 & 0 & \cdots & 0 \\ M_2 & M_1 & M_0 & 0 & \cdots & 0 \\ M_3 & M_2 & M_1 & M_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ M_r & M_{r-1} & M_{r-2} & M_{r-3} & \cdots & M_1 \end{vmatrix}$$

for all $r \in \mathbb{N} \setminus \{0\}$.

From its definition, it is clear that $\mathcal{M}$ is a triangle.

Furthermore, $\mathcal{M}$-transform of $u = (u_s) \in \omega$ is expressed with

$$v_r := (\mathcal{M}u)_r = \frac{1}{M_{r+2} - M_{r+1}} \sum_{s=0}^{r} M_s M_{r-s} u_s, \; (r \in \mathbb{N}). \tag{4}$$

In **(Erdem et al., 2024)**, the authors presented the Motzkin sequence spaces $c(\mathcal{M})$ and $c_0(\mathcal{M})$ as the domain of the Motzkin matrix by

$$c(\mathcal{M}) = \left\{ u = (u_s) \in \omega : \lim_{r\to\infty} \frac{1}{M_{r+2} - M_{r+1}} \sum_{s=0}^{r} M_s M_{r-s} u_s \text{ exists} \right\}$$

and

$$c_0(\mathcal{M}) = \left\{ u = (u_s) \in \omega : \lim_{r\to\infty} \frac{1}{M_{r+2} - M_{r+1}} \sum_{s=0}^{r} M_s M_{r-s} u_s = 0 \right\}$$

and they studied some algebraic and topological properties of newly described spaces.

3. Motivation of the study

Obtaining new normed or paranormed sequence spaces by using the special matrix and addressing some topics in these spaces (for example; completeness, inclusion relations, Schauder basis, duals, matrix transformations, compact operators and core theorems) have been seen as an important field of study since past years and many valuable researches have been carried out in this subject.

From the recent past to the present, a comprehensive literature stands out on the examination of paranormed sequence spaces created with the idea of the domain of triangular matrices in Maddox's spaces. In this context, Ahmad and Mursaleen **(1987)** introduced the sequence spaces

$$\Gamma(\Delta, \mathfrak{p}) = \{u \in \omega : \Delta u \in \Gamma(\mathfrak{p})\}$$

by the aid of the forward difference matrix Δ, where $\Gamma \in \{\ell_\infty, c, c_0\}$.

With similar idea, Altay and Başar **(2002; 2006a)** introduced the Riesz sequence spaces $r^t(\mathfrak{p})$, $r_0^t(\mathfrak{p})$, $r_c^t(\mathfrak{p})$ and $r_\infty^t(\mathfrak{p})$, which are expressed as $r^t(\mathfrak{p}) = (\ell(\mathfrak{p}))_{R^t}$, $r_0^t(\mathfrak{p}) = (c_0(\mathfrak{p}))_{R^t}$, $r_c^t(\mathfrak{p}) = (c(\mathfrak{p}))_{R^t}$ and $r_\infty^t(\mathfrak{p}) = (\ell_\infty(\mathfrak{p}))_{R^t}$, where R^t represents the Riesz matrix.

Similarly, Aydın and Başar **(2004; 2006)** presented and studied the spaces $c(A^r,\mathfrak{p}) = (c(\mathfrak{p}))_{A^r}$, $c_0(A^r,\mathfrak{p}) = (c_0(\mathfrak{p}))_{A^r}$ and $\ell(A^r,\mathfrak{p}) = \ell((q))_{A^r}$ and this work has contributed to broadening our perspective on the subject of paranormed sequence spaces.

Additionally, domains of Fibonacci and Catalan matrices in $\ell(\mathfrak{p})$ were subjected to a meticulous examination by Çapan and Başar **(2016)** and Alp **(2020)**, respectively.

In another study, Savaşçı and Başar **(2023)** gave the paranormed Cesàro sequence space $\ell(C_\alpha,\mathfrak{p})$ of order α as the domain of Cesàro mean C_α of order α.

One of the most recent and comprehensive study in this field is the article by Yaying and Başar **(2024)**, in which new paranormed sequence spaces created as domains of the Lambda–Pascal matrix on Maddox's spaces were obtained and their various properties were examined in depth.

The studies we have just mentioned are quite remarkable in terms of historically shaping our perspective on paranormed sequence spaces and related concepts.

Researchers who desire basic and more detailed information about aforementioned and related subjects are recommended to benefit from the sources **(Altay & Başar, 2006b; Candan & Güneş, 2015; Daglı & Yaying, 2023; Demiriz & Erdem, 2024, Ilkhan et al., 2019; Kara & Demiriz, 2015; Yaying, 2021)** and monographs **(Başar, 2022; Maddox, 1988; Mursaleen & Başar, 2020)**.

As a continuation of the study in **(Erdem et al., 2024)** and with a similar idea to the studies briefly summarized in this section, in this article we aimed to obtain new paranormed sequence spaces $c_0(\mathcal{M},\mathfrak{p})$ and $c(\mathcal{M},\mathfrak{p})$ by the aid of the Motzkin matrix and present various topological properties, duals and matrix transformations of these spaces.

4. Paranormed Motzkin sequence spaces

It is introduced two paranormed Motzkin sequence spaces $c_0(\mathcal{M},\mathfrak{p})$ and $c(\mathcal{M},\mathfrak{p})$ by using the conservative Motzkin matrix $\mathcal{M}$ in this part.

Now, we can describe paranormed the Motzkin sequence spaces $c_0(\mathcal{M},\mathfrak{p})$ and $c(\mathcal{M},\mathfrak{p})$ as follows:

$$c_0(\mathcal{M},\mathfrak{p}) = \left\{u=(u_s)\in w: \lim_{r\to\infty}\left|\frac{1}{M_{r+2}-M_{r+1}}\sum_{s=0}^{r} M_sM_{r-s}u_s\right|^{\mathfrak{p}_r}=0\right\},$$

$$c(\mathcal{M},\mathfrak{p}) = \left\{u=(u_s)\in w: \lim_{r\to\infty}\left|\frac{1}{M_{r+2}-M_{r+1}}\sum_{s=0}^{r} M_sM_{r-s}u_s - l\right|^{\mathfrak{p}_r}=0 \text{ for some } l\in\mathbb{R}\right\}.$$

In that case, $c_0(\mathcal{M},\mathfrak{p})$ and $c(\mathcal{M},\mathfrak{p})$ are given as

$$c_0(\mathcal{M},\mathfrak{p}) = [c_0(\mathfrak{p})]_{\mathcal{M}} \text{ and } c(\mathcal{M},\mathfrak{p}) = [c(\mathfrak{p})]_{\mathcal{M}},$$

with the notation (1).

For $\mathfrak{p}=(\mathfrak{p}_r)=e$, the foregoing sequence spaces are simplified to $c_0(\mathcal{M})$ and $c(\mathcal{M})$ which were previously presented by Erdem et al. **(2024)**.

Theorem 4.1. *The sequence spaces $c_0(\mathcal{M},\mathfrak{p})$ and $c(\mathcal{M},\mathfrak{p})$ are complete linear metric spaces paranormed with*

$$\mathfrak{f}(u) = \sup_{r\in\mathbb{N}}\left|\frac{1}{M_{r+2}-M_{r+1}}\sum_{s=0}^{r} M_sM_{r-s}u_s\right|^{\mathfrak{p}_r/\mathcal{Q}} \tag{5}$$

Proof. It is simply presented the proof for $c(\mathcal{M},\mathfrak{p})$ to avoid repeating the same statements.

$$\sup_{r\in\mathbb{N}}\left|\frac{1}{M_{r+2}-M_{r+1}}\sum_{s=0}^{r} M_sM_{r-s}(u_s+t_s)\right|^{\frac{\mathfrak{p}_r}{\mathcal{Q}}} \leq \sup_{r\in\mathbb{N}}\left|\frac{1}{M_{r+2}-M_{r+1}}\sum_{s=0}^{r} M_sM_{r-s}u_s\right|^{\frac{\mathfrak{p}_r}{\mathcal{Q}}} + \sup_{r\in\mathbb{N}}\left|\frac{1}{M_{r+2}-M_{r+1}}\sum_{s=0}^{r} M_sM_{r-s}u_s\right|^{\frac{\mathfrak{p}_r}{\mathcal{Q}}} \tag{6}$$

and

$$|\delta|^{\mathfrak{p}_r} \leq \max\{1,|\delta|^{\mathcal{Q}}\}, \tag{7}$$

it is reached the linearity of $c(\mathcal{M},\mathfrak{p})$ with respect to the coordinatewise addition and scalar multiplication for $u=(u_s), t=(t_s)\in c(\mathcal{M},\mathfrak{p})$ and $\delta\in\mathbb{R}$ (See **(Maddox, 1968; Madox, 1988)**). Moreover, it is seen the statements $\mathfrak{f}(\theta)=0$ and for all values of $u\in c(\mathcal{M},\mathfrak{p})$, $\mathfrak{f}(-u)=\mathfrak{f}(u)$ hold. From (6) and (7), it is achieved that $\mathfrak{f}$ is subadditive and

$$\mathfrak{f}(\delta u) \leq \max\{1,|\delta|\}\mathfrak{f}(u).$$

Consider $\{u^n\} \in \omega$ with $u^n \in c(\mathcal{M}, \mathfrak{p})$ such that $f(u^n - u) \to 0$ and (δ_n) with real terms such that $\delta_n \to \delta$. Sice $\mathfrak{f}$ is subadditive, the boundedness of $\{\mathfrak{f}(u^n)\}$ can be seen from the expression

$$\mathfrak{f}(u^n) \leq \mathfrak{f}(u) + \mathfrak{f}(u^n - u).$$

Thus, it is obtained that

$$\begin{aligned}\mathfrak{f}(\delta_n u^n - \delta u) &= \sup_{r\in\mathbb{N}} \left| \frac{1}{M_{r+2} - M_{r+1}} \sum_{s=0}^{r} M_s M_{r-s} (\delta_n u_s^n - \delta u_s) \right|^{\mathfrak{p}_r/\mathcal{Q}} \\ &\leq |\delta_n - \delta| \mathfrak{f}(u^n) + |\delta| \mathfrak{f}(u^n - u)\end{aligned}$$

which tends to zero as $n \to \infty$. In that case, the scalar multiplication is continuous. In all that has been said so far, $\mathfrak{f}$ is a paranorm on $c(\mathcal{M}, \mathfrak{p})$.

For the completeness of $c(\mathcal{M}, \mathfrak{p})$, let us choose any Cauchy sequence $\{u^i\}$ in $c(\mathcal{M}, \mathfrak{p})$ with $u^i = \{u_0^{(i)}, u_1^{(i)}, u_2^{(i)}, \cdots\}$. Therefore, for all $\epsilon > 0$, there exists $n_0(\epsilon) \in \mathbb{N}$ such that

$$\mathfrak{f}(u^i - u^s) < \frac{\epsilon}{2}$$

for all $i, s > n_0(\epsilon)$. For each constant $r \in \mathbb{N}$, we find from the definition of $\mathfrak{f}$ that

$$|(\mathcal{M}u^i)_r - (\mathcal{M}u^s)_r|^{\mathfrak{p}_r/\mathcal{Q}} \leq \sup_{r\in\mathbb{N}} |(\mathcal{M}u^i)_r - (\mathcal{M}u^s)_r|^{\mathfrak{p}_r/\mathcal{Q}} < \frac{\epsilon}{2} \tag{8}$$

for every $i, s > n_0(\epsilon)$. Consequently, for all fixed $r \in \mathbb{N}$, $\{(\mathcal{M}u^0)_r, (\mathcal{M}u^1)_r, (\mathcal{M}u^2)_r, \ldots\}$ is Cauchy sequence with real terms. From the completeness of $\mathbb{R}$, it is convergent. Thus $(\mathcal{M}u^i)_r \to (\mathcal{M}u)_r$ as $i \to \infty$.

We can describe the sequence $\{(\mathcal{M}u)_0, (\mathcal{M}u)_1, \ldots\}$ by using these infinitely limit points $(\mathcal{M}u)_0, (\mathcal{M}u)_1, (\mathcal{M}u)_2 \cdots$. It is obtained that

$$|(\mathcal{M}u^i)_r - (\mathcal{M}u)_r|^{\frac{\mathfrak{p}_r}{\mathcal{Q}}} \leq \frac{\epsilon}{2} \ (i > n_0(\epsilon)) \tag{9}$$

from (8) with $s \to \infty$ for any fixed $r \in \mathbb{N}$. Since $u^i = \left\{u_r^{(i)}\right\} \in c(\mathcal{M}, \mathfrak{p})$, there exists $r_0(\epsilon) \in \mathbb{N}$,

$$|(\mathcal{M}u^i)_r|^{\mathfrak{p}_r/\mathcal{Q}} < \frac{\epsilon}{2} \tag{10}$$

for all $r > r_0(\epsilon)$ and $i \in \mathbb{N}$. It is derived from (9) and (10) that

$$|(\mathcal{M}u)_r|^{\mathfrak{p}_r/\mathcal{Q}} \leq |(\mathcal{M}u)_r - (\mathcal{M}u^i)_r|^{\mathfrak{p}_r/\mathcal{Q}} + |(\mathcal{M}u^i)_r|^{\mathfrak{p}_r/\mathcal{Q}} < \epsilon$$

for every $r > r_0(\epsilon)$ and $i > n_0(\epsilon)$. This demonstrates that $u \in c(\mathcal{M}, \mathfrak{p})$. Consequently, $c(\mathcal{M}, \mathfrak{p})$ is complete.

Theorem 4.2. *The spaces* $c_0(\mathcal{M}, \mathfrak{p})$ *and* $c(\mathcal{M}, \mathfrak{p})$ *are linearly isomorphic to the spaces* $c_0(\mathfrak{p})$ *and* $c(\mathfrak{p})$, *respectively, where* $0 < \mathfrak{p}_r \leq S < \infty$.

Proof. In order not to repeat similar steps, it will be proven the theorem only for $c_0(\mathcal{M}, \mathfrak{p})$ and $c_0(\mathfrak{p})$. The goal is to demonstrate that there exist a linear bijection from $c_0(\mathcal{M}, \mathfrak{p})$ to $c_0(\mathfrak{p})$. To achieve this, we define a transformation $\mathcal{T}$ that maps $c_0(\mathcal{M}, \mathfrak{p})$ to $c_0(\mathfrak{p})$. This transformation is denoted by

$$u \mapsto \nu = \mathcal{T}u = \mathcal{M}u.$$

The linearity of $\mathcal{T}$ is evident, and it can be obtained that $\mathcal{T}$ is injective, because it follows that $\mathcal{T}u = \theta$ whenever $u = \theta$.

Let $\nu \in c_0(\mathcal{M}, \mathfrak{p})$ with (4), then we have

$$\begin{aligned}
\mathfrak{f}(u) &= \sup_{r\in\mathbb{N}} \left| \frac{1}{M_{r+2} - M_{r+1}} \sum_{s=0}^{r} M_s M_{r-s} u_s \right|^{\mathfrak{p}_r/\mathcal{Q}} \\
&= \sup_{r\in\mathbb{N}} \left| \frac{1}{M_{r+2} - M_{r+1}} \sum_{s=0}^{r} M_s M_{r-s} \sum_{i=0}^{s} (-1)^{s-i} \frac{M_{i+2} - M_{i+1}}{M_s} \pi_{s-i} \nu_i \right|^{\mathfrak{p}_r/\mathcal{Q}} \\
&= \sup_{r\in\mathbb{N}} \left| \sum_{s=0}^{r} \sigma_{rs} \nu_s \right|^{\mathfrak{p}_r/\mathcal{Q}} \\
&= \sup_{r\in\mathbb{N}} |\nu_r|^{\mathfrak{p}_r/\mathcal{Q}} < \infty,
\end{aligned}$$

where

$$\sigma_{rs} = \begin{cases} 1 & , \quad s = r, \\ 0 & , \quad s \neq r. \end{cases}$$

Thus, we get $u \in c_0(\mathcal{M}, \mathfrak{p})$, so $\mathcal{T}$ is surjective.

In conclusion, we deliver the Schauder basis for the spaces $c_0(\mathcal{M}, \mathfrak{p})$ and $c(\mathcal{M}, \mathfrak{p})$. Initially, we examine the definition of the Schauder basis. Consider a paranormed space $(\Gamma, \wp)$. There exists a singular sequence of scalars $(\hbar_s)$ for which $\wp(\nu - \sum_{s=0}^{r} \hbar_s \zeta_s) \to 0$ as $r \to \infty$ iff $(\zeta_s) \in \Gamma$ is deemed a basis of Γ.

Consider that $\Gamma \in \omega$ and D is a triangle. From Remark 2.4 of **(Jarrah and Malkowsky, 1990)**, Γ_D has a basis whenever Γ has a basis.

Theorem 4.3. *Set $\lambda_s = (\mathcal{M}u)_s$ for all $s \in \mathbb{N}$ and $0 < \mathfrak{p}_s \leq \mathcal{S} < \infty$. The sequence $b^{(s)} = \{b^{(s)}\}_{s\in\mathbb{N}}$ of the elements of the spaces $c_0(\mathcal{M},\mathfrak{p})$ and $c(\mathcal{M},\mathfrak{p})$ is described with*

$$b_r^{(s)} = \begin{cases} (-1)^{r-s}\dfrac{M_{s+2}-M_{s+1}}{M_r}\pi_{r-s} & , \quad 0 \leq s < r \\ 0 & , \quad otherwise \end{cases}$$

for $s \in \mathbb{N}$. In that case,

(a) $\{b^{(s)}\}_{s\in\mathbb{N}}$ *is a basis for the space $c_0(\mathcal{M},\mathfrak{p})$, and any $v \in c_0(\mathcal{M},\mathfrak{p})$ has a unique representation as*

$$v = \sum_s \lambda_s b^{(s)}.$$

(b) $\{e, b^{(1)}, b^{(2)}, \dots\}$ *is a basis for $c(\mathcal{M},\mathfrak{p})$, and any $v \in c(\mathcal{M},\mathfrak{p})$ has a unique representation as*

$$v = \tau e + \sum_s (\lambda_s - \tau) b^{(s)}$$

where $\tau = \lim_{s\to\infty} (\mathcal{M}u)_s$.

5. Duals

Current part of the paper aims to obtain the α-, β-, and γ-duals of our newly described spaces. It is referred to the collection of all finite subsets of $\mathbb{N}$ as $\mathcal{F}$.

Lemma 5.1. *Consider an infinite matrix $D = (d_{rs})$*

(i). $D \in (c_0(\mathfrak{p}): \ell(\mathfrak{q}))$ iff

$$\exists\, M > 1 \ni sup_{K\in\mathcal{F}} \sum_r \left|\sum_{s\in K} d_{rs} M^{-\frac{1}{\mathfrak{p}_s}}\right|^{\frac{1}{\mathfrak{q}_r}} < \infty \text{ for any } \mathfrak{q}_r \geq 1. \tag{15}$$

(ii). $D \in (c_0(\mathfrak{p}): c(\mathfrak{q}))$ iff

$$\exists\, M > 1 \ni sup_r \sum_s |d_{rs}|\, M^{-\frac{1}{\mathfrak{p}_s}} < \infty\,, \tag{16}$$

$$\exists (\psi_s) \in \mathbb{R}, \forall\, s \in \mathbb{N} \ni \lim_r |d_{rs} - \psi_s|^{\mathfrak{q}_r} = 0, \tag{17}$$

$$\forall\, L, \exists\, M > 1 \text{ and } \exists\, (\psi_s) \in \mathbb{R} \ni \sup_r L^{\frac{1}{\mathfrak{q}_r}} \sum_s |d_{rs} - \psi_s|\, M^{-\frac{1}{\mathfrak{p}_s}} < \infty. \tag{18}$$

(iii). $D \in (c_0(\mathfrak{p}): \ell_\infty(\mathfrak{q}))$ iff

$$\exists\, M > 1, \sup_r \left(\sum_{s=0}^{\infty} |d_{rs}|\, M^{-\frac{1}{\mathfrak{p}_s}} \right)^{\mathfrak{q}_r} < \infty. \tag{19}$$

Theorem 5.2. *Let $K \in \mathcal{F}$. Let us describe the sets $\aleph_1$ and $\aleph_2$ by the following way:*

$$\aleph_1 = \bigcup_{M>1} \left\{ \tau = (\tau_s) \in \omega : \sup_{K\in\mathcal{F}} \sum_{r=0}^{\infty} \left| \sum_{s\in K} (-1)^{r-s} \frac{\mathcal{M}_{s+2} - \mathcal{M}_{s+1}}{\mathcal{M}_r} \pi_{r-s} \tau_r M^{-1/\mathfrak{p}_s} \right| < \infty \right\},$$

$$\aleph_2 = \left\{ \tau = (\tau_s) \in \omega : \sum_{r=0}^{\infty} \left| \sum_{s=0}^{\infty} (-1)^{r-s} \frac{\mathcal{M}_{s+2} - \mathcal{M}_{s+1}}{\mathcal{M}_r} \pi_{r-s} \tau_r \right| < \infty \right\}.$$

In that case,

(i). $[c_0(\mathcal{M}, \mathfrak{p})]^{\alpha} = \aleph_1$,

(ii). $[c(\mathcal{M}, \mathfrak{p})]^{\alpha} = \aleph_1 \cap \aleph_2$.

Proof. To demonstrate the α-dual, consider the sequence $\tau = (\tau_r) \in \omega$. By applying the relationship (4), this is equivalent to the equation

$$\tau_r u_r = \sum_{s=0}^{r} (-1)^{r-s} \frac{\mathcal{M}_{s+2} - \mathcal{M}_{s+1}}{\mathcal{M}_r} \pi_{r-s} \tau_r \nu_s = (T\nu)_r,\ (r \in \mathbb{N}) \tag{20}$$

where the matrix $T = (t_{rs})$ defined by

$$t_{rs} = \begin{cases} (-1)^{r-s} \dfrac{\mathcal{M}_{s+2} - \mathcal{M}_{s+1}}{\mathcal{M}_r} \pi_{r-s} \tau_r & , \quad 0 \le s \le r, \\ 0 & , \quad \text{otherwise.} \end{cases}$$

By the relation (20), it is concluded that $\tau u = (\tau_r u_r) \in \ell_1$ whenever $u \in c_0(\mathcal{M}, \mathfrak{p})$ iff $T\nu \in \ell_1$ whenever $\nu \in c_0(\mathfrak{p})$. Thus, we obtain that $\tau = (\tau_r) \in [c_0(\mathcal{M}, \mathfrak{p})]^{\alpha}$ iff $T \in (c_0(\mathfrak{p}) : \ell_1)$. Consequently, from (15) with $\mathfrak{q}_r = 1$ for $r \in \mathbb{N}$, it is seen that $[c_0(\mathcal{M}, \mathfrak{p})]^{\alpha} = \aleph_1$.

Since the second part can be shown in a similar way, we omit the details.

Theorem 5.13. *Let $K \in \mathcal{F}$. Define the sets $\aleph_3, \ldots, \aleph_8$ as follows:*

$$\aleph_3 = \bigcup_{M>1}\left\{\tau=(\tau_s)\in\omega:\sup_{r\in\mathbb{N}}\sum_{s=0}^{\infty}\left|\sum_{i=s}^{r}(-1)^{s-i}\frac{M_{i+2}-M_{i+1}}{M_s}\pi_{s-i}\tau_i\right|M^{-1/\mathfrak{p}_s}<\infty\right\},$$
$$\aleph_4 = \left\{\tau=(\tau_s)\in\omega:\exists(\psi_s)\subset\mathbb{R}\ni\lim_{r}\left|\sum_{i=s}^{r}(-1)^{s-i}\frac{M_{i+2}-M_{i+1}}{M_s}\pi_{s-i}\tau_i-\psi_s\right|=0,\right\},$$
$$\aleph_5 = \bigcup_{M>1}\left\{\tau=(\tau_s)\in w:\exists(\psi_s)\subset\mathbb{R}\ni\sum_{s=0}^{\infty}\left|\sum_{i=s}^{r}(-1)^{s-i}\frac{M_{i+2}-M_{i+1}}{M_s}\pi_{s-i}\tau_i-\psi_s\right|M^{-1/\mathfrak{p}_s}<\infty\right\},$$
$$\aleph_6 = \left\{\tau=(\tau_s)\in\omega:\exists(\psi_s)\subset\mathbb{R}\ni\lim_{r}\left|\sum_{s=0}^{\infty}\sum_{i=s}^{r}(-1)^{s-i}\frac{M_{i+2}-M_{i+1}}{M_s}\pi_{s-i}\tau_i-\psi_s\right|<\infty\right\},$$
$$\aleph_7 = \bigcup_{M>1}\left\{\tau=(\tau_s)\in\omega:\sup_{r\in\mathbb{N}}\left(\sum_{s=0}^{\infty}\left|\sum_{i=s}^{r}(-1)^{s-i}\frac{M_{i+2}-M_{i+1}}{M_s}\pi_{s-i}\tau_i\right|M^{-1/\mathfrak{p}_s}\right)<\infty\right\},$$
$$\aleph_8 = \left\{\tau=(\tau_s)\in\omega:\sup_{r\in\mathbb{N}}\left|\sum_{s=0}^{\infty}\sum_{i=s}^{r}(-1)^{s-i}\frac{M_{i+2}-M_{i+1}}{M_s}\pi_{s-i}\tau_i\right|<\infty\right\}.$$

Then,

(i). $[c_0(\mathcal{M},\mathfrak{p})]^{\beta}=\aleph_3\cap\aleph_4\cap\aleph_5, [c_0(\mathcal{M},\mathfrak{p})]^{\gamma}=\aleph_7.$

(ii). $[c(\mathcal{M},\mathfrak{p})]^{\beta}=\aleph_3\cap\aleph_4\cap\aleph_5\cap\aleph_6, [c(\mathcal{M},\mathfrak{p})]^{\gamma}=\aleph_7\cap\aleph_8.$

Proof. The following equation is crucial for our proof:

$$\sum_{s=0}^{r}\tau_s u_s = \sum_{s=0}^{r}\left[\sum_{i=0}^{s}(-1)^{s-i}\frac{M_{i+2}-M_{i+1}}{M_s}\pi_{s-i}\nu_i\right]\tau_s$$
$$=\sum_{s=0}^{r}\left[\sum_{i=s}^{r}(-1)^{s-i}\frac{M_{i+2}-M_{i+1}}{M_s}\pi_{s-i}\tau_i\right]\nu_s = (O\nu)_r. \tag{21}$$

Here, $O=(o_{rs})$ is a matrix that is defined by

$$o_{rs}=\begin{cases}\sum_{i=s}^{r}(-1)^{s-i}\frac{M_{i+2}-M_{i+1}}{M_s}\pi_{s-i}\tau_i, & (0\le s\le r),\\ 0, & (s>r).\end{cases}$$

From (21), we can conclude that $\tau u=(\tau_s u_s)\in cs$ whenever $u=(u_s)\in c_0(\mathcal{M},\mathfrak{p})$ iff $O\nu\in c_0$ whenever $\nu=(\nu_s)\in c_0(\mathfrak{p})$. In other words, $\tau=(\tau_s)\in[c_0(\mathcal{M},\mathfrak{p})]^{\beta}$ iff $O\in(c_0(\mathfrak{p}):c)$. Thus, by using (16), (17), (18), and (21) with $\mathfrak{q}_r=1$ for all $r\in\mathbb{N}$, it is obtained that $[c_0(\mathcal{M},\mathfrak{p})]^{\beta}=\aleph_3\cap\aleph_4\cap\aleph_5$.

Since the other parts of the theorem can be shown in a similar way, it is passed the particulars.

6. Matrix mappings

This section characterized the classes $\left(\Gamma(\mathcal{M},\mathfrak{p}):\Psi\big((\mathfrak{q})\big)\right)$ of infinite matrices, where $\Gamma \in \{c_0, c\}$ and $\Psi \in \{\ell_1, c, \ell_\infty\}$.

Theorem 6.1. *Consider the sequence spaces $\Gamma \in \{c_0, c\}$ and $\Psi \in \{\ell_1, c, \ell_\infty\}$. In that case; $D = (d_{rs}) \in (\Gamma(\mathcal{M},\mathfrak{p}):\Psi)$ iff*

$$D_r \in \{\Gamma(\mathcal{M},\mathfrak{p})\}^\beta \text{ for all } r \in \mathbb{N} \tag{22}$$
$$B \in (\Gamma:\Psi) \tag{23}$$

for $B = (b_{rs})$ as $b_{rs} = \sum_{i=s}^{\infty} (-1)^{s-i} \frac{M_{i+2}-M_{i+1}}{M_s} \pi_{s-i} d_{ri}$.

Proof. Let $D = (d_{rs}) \in (\Gamma(\mathcal{M},\mathfrak{p}):\Psi)$ and $u \in \Gamma(\mathcal{M},\mathfrak{p})$. In that case, $D_r \in \{\Gamma(\mathcal{M},\mathfrak{p})\}^\beta$ for all $r \in \mathbb{N}$. For $\nu = \mathcal{M}u$, it is concluded from $D_r \in \{\Gamma(\mathcal{M},\mathfrak{p})\}^\beta$ that

$$(Du)_r = (B\nu)_r. \tag{24}$$

Therefore $B\nu \in \Psi$ for all $\nu \in \Gamma$. So,

$$B \in (\Gamma:\Psi).$$

Conversely, consider the satisfying of (22) and (23) and $u \in \Gamma(\mathcal{M},\mathfrak{p})$ with $\nu \in \Gamma$. In that case, Du exists. By using (24), we have $Du = B\nu$. Consequently, $D = (d_{rs}) \in (\Gamma(\mathcal{M},\mathfrak{p}):\Psi)$.

Let us state the followings which will be required to give some matrix classes related with the new described spaces.

$$\exists \psi \in \mathbb{R} \ni \lim_r \left|\sum_{s=0}^{\infty} d_{rs} - \psi\right|^{\mathfrak{q}_r} = 0. \tag{25}$$

$$\sum_r \left|\sum_{s=0}^{\infty} d_{rs}\right|^{\mathfrak{q}_r} < \infty. \tag{26}$$

$$\sup_r \left|\sum_{s=0}^{\infty} d_{rs}\right|^{\mathfrak{q}_r} < \infty. \tag{27}$$

Corollary 6.2. *Let $D = (d_{rs})$ be an infinite matrix. The following statements hold:*

(i). $D \in \left(c_0(\mathcal{M}, \mathfrak{p}) : \ell(\mathfrak{q}) \right)$ *iff* $\{d_{rs}\}_{s\in\mathbb{N}} \in \{c_0(\mathcal{M}, \mathfrak{p})\}^\beta$ *for all* $r \in \mathbb{N}$ *and* (15) *holds with* b_{rs} *in place of* d_{rs} *for* $\mathfrak{q}_r = 1$.

(ii). $D \in \left(c_0(\mathcal{M}, \mathfrak{p}) : c(\mathfrak{q}) \right)$ *iff* $\{d_{rs}\}_{s\in\mathbb{N}} \in \{c_0(\mathcal{M}, \mathfrak{p})\}^\beta$ *for all* $r \in \mathbb{N}$ *and* (16), (17), *and* (18) *hold with* b_{rs} *in place of* d_{rs} *for* $\mathfrak{q}_r = 1$.

(iii). $D \in \left(c_0(\mathcal{M}, \mathfrak{p}) : \ell_\infty(\mathfrak{q}) \right)$ *iff* $\{d_{rs}\}_{s\in\mathbb{N}} \in \{c_0(\mathcal{M}, \mathfrak{p})\}^\beta$ *for all* $r \in \mathbb{N}$ *and* (19) *holds with* b_{rs} *in place of* d_{rs} *for* $\mathfrak{q}_r = 1$.

Corollary 6.3. *Let $D = (d_{rs})$ be an infinite matrix. The following statements hold:*

(i). $D \in \left(c(\mathcal{M}, \mathfrak{p}) : \ell(\mathfrak{q}) \right)$ *iff* $\{d_{rs}\}_{s\in\mathbb{N}} \in \{c(\mathcal{M}, \mathfrak{p})\}^\beta$ *for all* $r \in \mathbb{N}$ *and* (15) *and* (26) *holds with* b_{rs} *in place of* d_{rs} *for* $\mathfrak{q}_r = 1$.

(ii). $D \in \left(c(\mathcal{M}, \mathfrak{p}) : c(\mathfrak{q}) \right)$ *iff* $\{d_{rs}\}_{s\in\mathbb{N}} \in \{c(\mathcal{M}, \mathfrak{p})\}^\beta$ *for all* $r \in \mathbb{N}$ *and* (16), (17), (18) *and* (25) *hold with* b_{rs} *in place of* d_{rs} *for* $\mathfrak{q}_r = 1$.

(iii). $D \in \left(c(\mathcal{M}, \mathfrak{p}) : \ell_\infty(\mathfrak{q}) \right)$ *iff* $\{d_{rs}\}_{s\in\mathbb{N}} \in \{c(\mathcal{M}, \mathfrak{p})\}^\beta$ *for all* $r \in \mathbb{N}$ *and* (19) *and* (27) *holds with* b_{rs} *in place of* d_{rs} *for* $\mathfrak{q}_r = 1$.

7. Conclusion

As a continuation of the article presented by Erdem et al. (2024), this study focuses on a comprehensive exploration of the domains of the Motzkin matrix $\mathcal{M}$ on the Maddox spaces $c_0(\mathfrak{p})$ and $c(\mathfrak{p})$. We also devote our efforts to uncovering the α-, β- and γ-duals of the newly obtained paranormed Motzkin sequence spaces and the necessary and sufficient conditions for matrix transformations between these spaces and the classical sequence spaces.

The new normed or paranormed sequence spaces obtained by using Motzkin or another infinite matrix and some of their algebraic and topological properties are the main targets of our future work.

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