Dera Natung Government College Research Journal
Volume 9 Issue 1, pp. 67-75, January-December 2024

ISSN (Print): 2456-8228
ISSN (Online): 2583-5483

Research Article

On almost GO-Menger spaces

Susmita Sarkar[a], **Prasenjit Bal**[b,*]

[a,b]Department of Mathematics, ICFAI University Tripura, Kamalghat-799210, India.

Cite as: Sarkar, S., & Bal, P. (2024). On almost GO-Menger spaces, Dera Natung Government College Research Journal, 9, 67-75.

https://doi.org/10.56405/dngcrj.2024.09.01.06

Received on: 17.07.2024,

Revised on: 27.09.2024,

Accepted on: 28.09.2024,

Available online: 30.12.2024.

*Corresponding Author: **Prasenjit Bal** (balprasenjit177@gmail.com)

Abstract: By employing g-open sets, we present the concept of almost GO-Menger space in this article. After that, the nature of almost GO-Menger space is compared to GO-Menger space, and some fundamental topological aspects of such spaces are examined. Additionally, a study of this space's quasi-irresolute image and an investigation into possible connections with some selection principles are conducted.

Keywords: Selection Principles, Menger space, GO-Menger space, Almost GO-Menger space.

MSC Subject classification: 54C10, 54D35, 54A20.

1. Introduction

For most of the topologists of the world most fascinating covering attributes are compactness, Lindelöfness, and Mengerness. Karl Menger introduced the concept of Mengerness, a sequential covering feature, in 1924 **(Menger, 1924)**. In literature, there are essentially two ways to generalise these covering features. Some generalisations are made using different selection principles (see **(Bal & Bhowmik, 2017; Bal et al., 2018; Bal & Kočinac, 2020)**), while others are made using different covering sets (see **(Menger, 1924; Rajesh &Vijayabharati, 2014)**). We apply both types of variations on the Mengerness property concurrently to bring about more intriguing extensions.

In the year 1970, Norman Levine presented the idea of generalised closed sets of a topological space **(Levine, 1970)**. A subset A of a topological space (X,τ) is called g-closed if $A \subseteq G \in \tau$ implies that $\bar{A} \subseteq G$**(Levine, 1970)**. Dunham **(1977)**and Dunham et al. **(1980)** conducted in-depth research on the characteristics of g-closed sets. g-open sets were described in those studies as the complement of g-closed sets.

Without regard to g-closed sets, Bal et al. established another equivalent concept of g-open sets in **(Sarkar et al., 2023)**. A subset A of a topological space X is called g-open set, if $V \subseteq int(A)$ whenever $V \subseteq A$ and for all closed set V**(Sarkar et al., 2023)**. α-open sets **(Njastad, 1965)**, b-open sets **(Menger, 1924)**, θ-open sets **(Veličko, 1966)** and various generalisations of open sets are also being studied by many mathematicians. However, Dunham's **(1977, 1980)** findings make the g-opens sets the most intriguing extension of open sets. Using the g-open sets as a tool, Balachandran et al. **(Balachandran, 1991)** proposed the GO-compactness, Bal et al. **(Sarkar et al., 2023)** proposed GO-Lindelöfness, and GO-Mengerness and thoroughly investigated their features . In a similar manner, we used g-open sets to introduced almost GO-Mengerness and looked at some of its topological attributes in this study.

GO-Menger space is a study of sequential covering properties which can further be used in the study of selection principles and topological games.

2. Literature Review

For the readers' advantage, a few fundamental topics are discussed in this section.

For a topological space (X, τ), a collection $\boldsymbol{A}$ of subsets of X is called a cover for the space X if $\bigcup \boldsymbol{U} = X$. If the collection is a collection of open subsets of X then it is called an open cover **(Engelking, 1989)**. Suppose, $\boldsymbol{O}$ denotes the family of all open covers of X. Then

Definition 2.1. (Kočinac, 2015) ***$S_{\{fin\}}(\boldsymbol{A}, \boldsymbol{B})$ denotes the following selection principle*** *: for each sequence $\{A_n : n \in \mathbb{N}\}$ of elements of $\boldsymbol{A}$ there is a sequence $\{B_n : n \in \mathbb{N}\}$ of finite sets such that for each $n \in \mathbb{N}, B_n \subseteq A_n$ and $\bigcup_{n \in \mathbb{N}} B_n \epsilon \boldsymbol{B}$, where $\boldsymbol{A}$ and $\boldsymbol{B}$ are families of subsets of a space X or collection of families of subsets of a space X.*

Definition 2.2. **(Menger, 1924)** *A space X which satisfies the selection property $S_{fin}(\boldsymbol{O}, \boldsymbol{O})$ is called an Menger space.*

A subset A of a topological space (X, τ) is called g-closed if $A \subseteq G \in \tau$ implies that $\bar{A} \subseteq G$ **(Levine, 1970)**. $gCl(A)$ denotes the g-closer of a set $A \subseteq X$ and is defined as the smallest g-closed set containing A. Arbitrary union of g-closed sets is a g-open set. Levine **(1970)** studied g-open sets as the complement of g-closed sets. Sarkar et al. **(2023)** proposed an alternative equivalent definition of g-open sets. A subset A of a topological space X is called g-open set, if $V \subseteq int(A)$ whenever $V \subseteq A$ and for all closed set V **(Sarkar et al., 2023)**. A set $A \subseteq X$ is called a g-regular subset if A is both g-open and g-closed.

Throughout the paper $\boldsymbol{R}(\boldsymbol{X})$ will denote the collection of all g-regular subsets of $X, GO(X)$ will denote the collection of all g-open subsets of X, $\boldsymbol{GO}(\boldsymbol{X})$ will denote the collection of all g-open covers of X.

Definition 2.3. **(Sarkar et al., 2023)** *A space X which satisfies the selection property $S_{fin}(\boldsymbol{GO}, \boldsymbol{GO})$ is called an GO-Menger space.*

Example 2.4. **(Sarkar et al., 2023)** Let $X = \mathbb{N}$ equipped with the discrete topology τ_δ and $\{\boldsymbol{U}_n : n \in \mathbb{N}\}$ be an arbitrary sequence of g-open covers of X. For each $n \in \mathbb{N}$, there exists a $U_n \in \boldsymbol{U}_n$ such that $n \in \mathbb{N}$. So for each $n \in \mathbb{N}$, if we choose $\boldsymbol{V}_n = \{U_n\}$. Then $\boldsymbol{V}_n \subseteq \boldsymbol{U}_n$ is a finite subset for each $n \in \mathbb{N}$. Also $\cup_{n\in\mathbb{N}} \boldsymbol{V}_n$ froms a g-open cover of X. So the space is GO-Menger.

Definition 2.5. **(Balachandran et al., 1991)** *A map $f : (X, \tau) \to (Y, \sigma)$ is called*
a) ***{ g-continuous}*** *if $f^{-1}(A) \in GO(X)$ for all $A \in \sigma$.*
b) *{**gc-irresolute**} if $f^{-1}(A) \in GO(X)$ for all $A \in GO(Y)$.*

No specific separation axiom is assumed for this paper unless otherwise stated and for the usual notions of topology we follow **(Engelking, 1989)**.

3. Almost GO-Menger Spaces

Although GO-Menger space weaken's the concept of Menger property, we want to search for some other covering property which weaker than GO-Menger space but stronger than Menger space. With this aim we introduce the following definition.

Definition 3.1. *Let $\boldsymbol{GO}(X)$ denotes the collection of all g-open covers of X. A topological space X is said to be almost GO-Menger space if for every sequence $\{\boldsymbol{U}_n : n \in \mathbb{N}\}$, where $\boldsymbol{U}_n \in \boldsymbol{GO}(X)$ we can find a sequence $\{\boldsymbol{V}_n : n \in \mathbb{N}\}$ such that $\boldsymbol{V}_n \subseteq \boldsymbol{U}_n$ is finite for all $n \in \mathbb{N}$ and $\cup_{\{n\in\mathbb{N}\}} \{\cup \{gCl(V) : V \in \boldsymbol{V}_n\}\} = X$.*

Definition 3.2. *In a topological space X, $A \subseteq X$ will be called a g-dense subset if $gCl(A) = X$.*

Proposition 3.3. *If a topological space X contains a g-dense subset which is GO-Menger in X, then X is almost GO-Menger space.*

Proof. Let A be a g-dense subset of X which is also GO-Menger in X and suppose $\{ \boldsymbol{U}_n : n \in \mathbb{N}\}$ is a sequence of covers of A such that $\boldsymbol{U}_n \in \boldsymbol{GO}(X)$ for each $n \in \mathbb{N}$.

Since A is GO-Menger in X, there exists a sequence $\{ \boldsymbol{V}_n : n \in \mathbb{N}\}$ such that $\boldsymbol{V}_n \subseteq \boldsymbol{U}_n$ is finite and $A \subseteq \cup_{\{n\in\mathbb{N}\}} \{\cup \{ V : V \in \boldsymbol{V}_n \}\}$.

But $A \subseteq \cup_{\{n\in\mathbb{N}\}} \{\cup \{V : V \in \boldsymbol{V}_n\}\} \subseteq \cup_{\{n\in\mathbb{N}\}} \{ \cup \{gCl(V): V \in \boldsymbol{V}_n\} \}$.

Taking gCl we have,

$X = gCl(A) \subseteq \cup_{\{n\in\mathbb{N}\}} \{ \cup \{gCl(V): V \in \boldsymbol{V}_n\} \}$ $[\because A \text{ is } g-dense,\ gCl(A) = X]$.Thus $\cup_{\{n\in\mathbb{N}\}} \{\{ gCl(V) : V \in \boldsymbol{V}_n \}\} = X$. Hence the proposition.

Definition 3.4. *Let $GO(X)$ denotes the collection of a g-open sets of a space X. A space X is g-regular if for each g-closed set A and a point $x \notin A$ there exists $U, V \in GO(X)$ such that $x \in U, A \subseteq V$ and $U \cap V = \emptyset$.*

Theorem 3.5. *The following statements are equivalent.*

(i) *X is a g-regular space.*

(ii) *For each $G \in GO(X)$ with $x \in G, \exists H \in GO(X)$ such that $x \in H \subseteq gCl(H) \subseteq G$.*

Proof. (i) $\Rightarrow$ (ii)

Let X is g-regular and $G \in GO(X)$ with $x \in G$. So, $X \setminus G = F$(say) is a closed set with $x \notin F$.

Therefore by g-regularity there exists $H, V \in GO(X)$ such that

$x \in H, F \subseteq V \text{ and } H \cap V = \emptyset$

$\Longrightarrow H \cap F = \emptyset$

$\Longrightarrow F \subseteq X \setminus H$

$\Longrightarrow X \setminus G \subseteq X \setminus H$

$\Longrightarrow H \subseteq G$

Moreover, $H \cap V = \emptyset$

$\Longrightarrow H \subseteq X \setminus V \subseteq X \setminus F = X \setminus (X \setminus G) = G$

$\Longrightarrow H \subseteq X \setminus V \subseteq G$ *but*, $X \setminus V$ is a g-closed set containing H

$\Longrightarrow gCl(H) \subseteq gCl(X \setminus V)$

$\Longrightarrow gCl(H) \subseteq X \setminus V \subseteq G$.

Therefore, $x \in H \subseteq gCl(H) \subseteq G$.

(ii) $\Rightarrow$ (i)

Let statement (ii) holds. Let $x \in X$ and F be any g-closed set such that $x \notin F$. Therefore $x \in X \setminus F = G$ (say) and G is a g-open set. Therefore by the given condition there exist $H \in GO(X)$ such that

$x \in H \subseteq gCl(H) \subseteq G.$

$\Longrightarrow x \in H \text{ and } X \setminus G \subseteq X \setminus gCl(H).$

$\Longrightarrow x \in H \text{ and } X \setminus (X \setminus F) \subseteq V = X \setminus gCl(H) \text{ (say)}.$

$\Longrightarrow x \in H \text{ and } F \subseteq V \text{ where } H, V \in GO(X).$

Also, $H \subseteq gCl(H)$

$\Longrightarrow H \cap (X \setminus gCl(H)) = \emptyset$

$\Longrightarrow H \cap V = \emptyset$

Therefore X is a g-regular space.

Theorem 3.6. *A GO-Menger space is always an almost GO-Menger space.*

Proof. The proof follows directly from the definition and the fact that $gCl(V) \supseteq V$ for all $V \subseteq X$.

Example 3.7. There exists a topological space which is neither GO-Menger nor almost GO-Menger.

Let $X = [0, \infty)$ and $\boldsymbol{B} = \{B_n = [0, n) : n \in \mathbb{N}\} \cup \{\emptyset\}$ is a base for the topology τ on X. Now we want to show that in the space X, $A \subseteq X$ which does not have the supremum element in X is g-closed and if A has a supremum in X, then A is not g-closed. Since X is bounded below, every element of X will have a infimum in X.

Suppose A has a supremum a_{sup} in X then $A \subseteq [0, a_{sup} + 1) \in \tau$. But $\bar{A} = [a_{inf}, \infty) \subsetneq [0, a_{sup} + 1)$, ($a_{inf}$ is the infimum of A.)

Therefore A is not a g-closed set.

Now, suppose that A do not have supremum in X then X is the only open set containing A and $\bar{A} \subseteq X$. i.e. A is g-closed.

In the same space X, every single ton $\{a\}$ is a g-open set. Because $X \setminus \{a\}$ does not have any supremum.

Consider the cover $\boldsymbol{U} = \{\{x\} : x \in X\}$ and the sequence $\{\boldsymbol{U}_n = U : n \in \mathbb{N}\}$ of g-open covers of X. Now if we consider any finite subset $\boldsymbol{V}_n of\ \boldsymbol{U}_n$ for each $n \in \mathbb{N}$ then $\cup_{\{n \in \mathbb{N}\}} \boldsymbol{V}_n$ will not be a cover of X Since countable union of finite union of singletons is countable.

Therefore X is not GO-Menger.

Now, for the same sequence $\{\boldsymbol{U}_n : n \in \mathbb{N}\}$ of open covers suppose we choose the sequence $\{\boldsymbol{V}_n : n \in \mathbb{N}\}$ such that $\boldsymbol{V}_n \subseteq \boldsymbol{U}_n$ is finite for each $n \in \mathbb{N}$.

$V_{\boldsymbol{V}_n} \in \boldsymbol{V}_n$ is singleton set for each $n \in \mathbb{N}$. Since $V_{\boldsymbol{V}_n} \cup [1, \infty)$ is a g-closed set containing $V_{\boldsymbol{V}_n}, \therefore gCl(V_{\boldsymbol{V}_n}) \subseteq V_{\boldsymbol{V}_n} \cup [1, \infty)$ for each $n \in \mathbb{N}$.

$$\Rightarrow \bigcup_{\{V\in \boldsymbol{V}_n\}} gCl(V_{V_n}) \subseteq \left(\bigcup_{\{V_{V_n}\in \boldsymbol{V}_n\}} V_{V_n}\right) \cup [1,\infty) \ for\ each\ n\in \mathbb{N}.$$

$$\Rightarrow \bigcup_{\{n\in\mathbb{N}\}} \left\{\bigcup_{\{V\in \boldsymbol{V}_n\}} gCl(V_{V_n})\right\} = \left\{\bigcup_{\{n\in\mathbb{N}\}}\left(\bigcup_{\{V_{V_n}\in \boldsymbol{V}_n\}} V_{V_n}\right)\right\} \cup [1,\infty) \neq X$$

Because $\cup_{\{n\in\mathbb{N}\}} \ (\cup_{\{V_{V_n}\in \boldsymbol{V}_n\}} V_{V_n})$ is a countable union of finite union of singletons which is a countable set and it cannot cover the uncountable set $[0,1)$.

Open Problem 3.8. *Does there exists an almost GO-Menger space which is not a GO-Menger space?*

Theorem 3.9. *A g-regular almost-g-Menger space is a g-Menger space.*

Proof. Let $\{\boldsymbol{U}_n : n\in\mathbb{N}\}$ be a sequence such that $\boldsymbol{U}_n \in \boldsymbol{GO}(X)$ for each $n\in\mathbb{N}$ in a topological space (X,τ). By theorem **(Andrijevic, 1996)**, for each $n\in\mathbb{N}$, $\forall\, U\in \boldsymbol{U}_n$ such that $a\in U$ there exists a $\boldsymbol{V}_a \in GO(X)$ such that $a\in V_a \subseteq gCl(V_a) \subseteq U$.

Suppose $\boldsymbol{M}_U = \{V_a : a\in U\}$ for each $U\in U_n$ and $n\in\mathbb{N}$ and assume that $\boldsymbol{V}_n = \cup_{\{U\in U_n\}} \boldsymbol{M}_U \ for\ each\ n\in\mathbb{N}$.

$$\Rightarrow \boldsymbol{V}_n = \cup_{\{U\in \boldsymbol{U}_n\}} \{V_a : a\in U\} \ for\ each\ n\in\mathbb{N}\}.$$

Thus $\boldsymbol{V}_n \in \boldsymbol{GO}(X)$ for each $n\in\mathbb{N}$. Moreover $\boldsymbol{V}_n' = \{gCl(V) : V\in \boldsymbol{V}_n\}$ is a refinement of $\boldsymbol{U}_n$ for each $n\in\mathbb{N}$.

But $\{\boldsymbol{V}_n : n\in\mathbb{N}\}$ is a sequence such that $\boldsymbol{V}_n \in \boldsymbol{GO}(X)$ for each $n\in\mathbb{N}$ and (X,τ) is almost GO-Menger. Therefore there exists a sequence $\{\boldsymbol{W}_n : n\in\mathbb{N}\}$ such that $\boldsymbol{W}_n \subseteq \boldsymbol{V}_n$ is finite for all $n\in\mathbb{N}$ and $\cup_{\{n\in\mathbb{N}\}} \{\cup \{gCl(W) : W\in \boldsymbol{W}_n\}\} = X$.

Now for each $n\in\mathbb{N}$ and $W\in \boldsymbol{W}_n$ we can choose $U_W \in \boldsymbol{U}_n$ such that $W\subseteq gCl(W)\subseteq U_W$. Let $\boldsymbol{U}_n' = \{U_W : W\in \boldsymbol{W}_n\}$. So, $\{\boldsymbol{U}_n' : n\in\mathbb{N}\}$ is a sequence such that $\boldsymbol{U}_n' \subseteq \boldsymbol{U}_n$ is finite for each $n\in\mathbb{N}$.

We have to show that $\cup_{\{n\in\mathbb{N}\}} \{\cup \boldsymbol{U}_n'\} = X$.

Let $x\in X$ be arbitrary. Since $\cup_{\{n\in\mathbb{N}\}} \{\cup \{gCl(W) : W\in \boldsymbol{W}_n\}\} = X$, there exists a $m\in\mathbb{N}$ and $W\in \boldsymbol{W}_M$ such that $x\in gCl((W)$. By the construction, there exists $U_W\in \boldsymbol{U}_M'$ such that $x\in gCl(W)\subseteq U_W$. Therefore $\cup_{\{n\in\mathbb{N}\}} \{\cup \boldsymbol{U}_n'\} = X$. Hence X is a GO-Menger space.

Definition 3.10. *A map* $f : (X,\tau) \to (Y,\sigma)$ *is called quasi-irresolute if* $f^{-1}(A)\in \boldsymbol{R}(X)$ *for all* $A\in \boldsymbol{R}(Y)$.

Theorem 3.11. *Let (X, τ) be an almost GO-Menger space and (Y, σ) a topological space. If $f : X \to Y$ is a quasi-irresolute surjection, then (Y, σ) is an almost GO-Menger space.*

Proof. Let $\{\boldsymbol{U}_n : n \in \mathbb{N}\}$ be a sequence of covers of $\boldsymbol{Y}$ by g-regular sets. Let $\boldsymbol{U}'_n = \{f^{-1}(U): U \subseteq \boldsymbol{U}_n\}$ for each $n \in \mathbb{N}$ is a sequence of g-regular covers of X. Since $'f'$ is a quasi-irresolute surjection and X is an almost GO-Menger space then there exists a sequence $\{\boldsymbol{V}_n : n \in \mathbb{N}\}$ such that for every $n \in \mathbb{N}$, $\boldsymbol{V}_n$ is a finite subset of $\boldsymbol{U}'_n$ and $\cup_{\{n\in\mathbb{N}\}} \boldsymbol{V}_n$ is a cover of X, $\cup_{\{n\in \mathbb{N}\}} \{\cup \{ gCl_X(V) = V : V \in \boldsymbol{V}_n\}\} = X$.

For each $n \in \mathbb{N}$, $V \in \boldsymbol{V}_n$ we can choose a $U_V \in \boldsymbol{U}_n$ such that, $V = f^{-1}(U_V)$. Now construct a sequence $\boldsymbol{W}_n = \{ gCl_Y(U_V) = U_V : V \in \boldsymbol{V}_n \}$ by g-regular sets of $\boldsymbol{U}_n$.

Now, if $y = f(x) \in Y$ then there exists a $V \in \boldsymbol{V}_n$, for each $n \in \mathbb{N}$ such that $x \in V$. Since $V = f^{-1}(U_V)$. Therefore $x \in f^{-1}(U_V)$.

$\Longrightarrow y = f(x) \in U_V \in \boldsymbol{W}_n$.

Therefore $\cup_{\{n\in \mathbb{N}\}} \boldsymbol{W}_n$ is a cover of Y.

Hence (Y, σ) is almost GO-Menger space.

Definition 3.12. *A g-open cover $\boldsymbol{U}$ is a gω-cover of a space X if $\boldsymbol{X}$ does not belong to $\boldsymbol{U}$ and every finite subset of $\boldsymbol{X}$ is contained in a member of $\boldsymbol{U}$.*

Let $\boldsymbol{G}\Omega$ denoted be the collection of all gω-covers for a space X.

Theorem 3.13. *For a space X the following are equivalent:*

(i) *X is GO-Menger.*

(ii) *X satisfies $S_{fin}(\boldsymbol{G}\Omega, \boldsymbol{G}O)$.*

Proof. (i) ⇒ (ii)

Every gω-cover of X is a g-open cover for X, which implies that every GO-Menger space satisfies the selection principle $S_{fin}(\boldsymbol{G}\Omega, \boldsymbol{G}O)$.

(ii) ⇒ (i)

Let $\{\boldsymbol{U}_n: n \in \mathbb{N}\}$ be a sequence of g-open covers of X. Partition $\mathbb{N}$ into pairwise disjoint infinite subsets $\{N_i: \mathbb{N} = N_1 \cup N_2 \cup \ldots \cup N_m \cup \ldots\}$.

For each $n \in \mathbb{N}$, $\boldsymbol{V}_n$ be the set of all elements of the form $U_{n_1} \cup U_{n_2} \cup \ldots \cup U_{n_k}$, $n_1 \leq n_2 \leq \ldots \leq n_k$, $n_i \in N_n$, $U_{n_i} \in \boldsymbol{U}_n$, $i \leq k$, $k \in \mathbb{N}$ which are not equal to X. Then every $\boldsymbol{V}_n$ is a gω-cover of X. Now X satisfies $S_{fin}(\boldsymbol{G}\Omega, \boldsymbol{G}O)$ so there exist a sequence $\{\boldsymbol{W}_n: n \in \mathbb{N}\}$ such that $\boldsymbol{W}_n \subseteq \boldsymbol{V}_n$ is finite and $\cup_{\{n\in\mathbb{N}\}} \{\cup_{\{W\in\boldsymbol{W}_n\}} W\} = X$.

Suppose $\boldsymbol{W}_n = \{ W_n^1, W_n^2, \ldots, W_n^m \}$ where each $W_n^i = U_n^{1^{i_n}} \cup U_n^{2^{i_n}} \cup \ldots \cup U_n^{k^{i_n}}$. So in this way we get finite subsets of $\boldsymbol{U}_p$ for some $p \in \mathbb{N}$ which cover X. If there are no elements from some $\boldsymbol{U}_p$ chosen in this way, then we put $\boldsymbol{W_p} = \emptyset$.

This gives that X is GO-Menger.

4. Conclusion

Almost GO-Menger space is a weaker version of GO-Menger space which is preserved under quasi-irresolute maps. A g-regular almost GO-Menger space is equivalent to GO-Menger space. A space containing a g-dense, GO-Menger subset is always almost GO-Menger space. GO-Menger space can also be characterise in terms of selection of principles .

Acknowledgment: The authors thank the reviewers for carefully reading the article and suggesting necessary changes that has improved the presentation of the article.

Availability of Data and Materials: Not applicable.

Conflicts of Interest: Authors declare no conflicts of interest.

Funding: Not applicable.

Authors' Contributions: All the authors contributed equally to the article. All the authors read and approved the final version.

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