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Research Article

Compactness via Hausdorff measure of non-compactness and some properties on Tetranacci sequence spaces

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Abstract: The characterization of compact operators on BK-spaces, which is the basis of this research, makes use of the Hausdorff measure of non-compactness. In this study, the compactness criteria of matrix operators defined on BK-spaces $\ell_p(\mathcal{T})$ and $\ell_\infty(\mathcal{T})$ which are the domains of the regular infinite Tetranacci matrix obtained by using the Tetranacci number sequence in ℓ_p and ℓ_∞, respectively, are investigated by using Hausdorff measure of non-compactness and some properties of $\ell_p(\mathcal{T})$ are examined.

Keywords: Tetranacci numbers, Sequence spaces, Hausdorff measure of non-compactness, Compact operators.

MSC 2020: 11B83, 46A45, 46B45, 47B07, 47B37.

1. Introduction

The symbol ω refers to the linear space of all sequences with real elements and any linear subspace of ω is called as a sequence space. Well-known sequence spaces can be instantiated as the space c of convergent sequences, the space c_0 of null sequences, the space ℓ_∞ of bounded sequences and the space ℓ_p of absolutely p-summable sequences. The aforementioned spaces are Banach spaces due to the norms $\|y\|_{\ell_\infty} = \|y\|_c = \|y\|_{c_0} = \sup_{k\in\mathbb{N}}|y_k|$ and $\|y\|_{\ell_p} = (\sum_k |y_k|^p)^{1/p}$, where $1 \le p < \infty$, the notation $\sum_k$ means $\sum_{k=1}^{\infty}$ and $\mathbb{N} = \{1,2,3,\dots\}$. Furthermore, the acronyms cs, cs_0 and bs refer the all convergent, null, and bounded series' spaces, respectively. A Banach space wherein each coordinate functional f_k, defined by $f_k(y) = y_k$, exhibits continuity, is known as a BK-space.

For an infinite matrix $D = (d_{nk})_{n,k\in\mathbb{N}}$ with real terms, D_n represents the n^{th} row of D. The D-transform of $y = (y_k) \in \omega$, denoted with $Dy = (Dy)_{n\in\mathbb{N}}$, is described as $\sum_{k=1}^{\infty} d_{nk} y_k$ if the series converges. A matrix that transforms a convergent sequence into a convergent sequence while preserving limit is called as regular.

Let $\Pi, \Theta \subset \omega$. If $Dy \in \Theta$ for all $y \in \Pi$, a matrix D is named as matrix transformation between the spaces Π and Θ and the family of matrix transformations between Π and Θ is indicated by $(\Pi:\Theta)$. Furthermore, the domain Θ_D of D on the space Θ is describes as

$$\Theta_D = \{y \in \omega : Dy \in \Theta\} \tag{1}$$

and this is a sequence space, too. We can see in Wilansky (1984) that if Θ is BK-space and D is triangle, in that case Θ_D is also BK-space with the norm described as $\|y\|_{\Theta_D} = \|Dy\|_{\Theta}$. BK-spaces (or Banach spaces) formulated through the application of the Euler, Cesàro, Riesz, Difference, Λ matrices (Altay & Başar, 2002, 2006a, 2006b; Başar & Altay, 2022; Mursaleen & Noman, 2010a; Ng & Lee, 1978; Sengönül & Başar, 2005), among others, represent some of the seminal works in this area. More detailed information on this topic can be found in sources (Başar, 2002; Mursaleen & Başar, 2020).

The notion of defining sequence spaces with triangular matrices obtained by using special integer sequences is based on the study of Kara and Başarır (2012) and the authors described Fibonacci sequence spaces with the help of Fibonacci number sequence. Then, as an application of summability theory to sequence spaces, some algebraic, topological and geometric properties of new sequence spaces obtained in studies carried out with similar logic using Tribonacci, Mersenne, Motzkin, Padovan, Catalan, Schröder, Bell, Leonardo, Lucas and some other sequences (Candan, 2012, 2013, 2015; Dağlı, 2023; Dağlı & Yaying, 2023; Demiriz & Erdem, 2024; Ellidokuzoğlu & Demiriz, 2018; Erdem et al., 2024; Erdem, 2024a, 2024b; İlkhan & Kara, 2021; Kara, 2013; Karakaş, 2023; Karakaş & Karabudak, 2017; Yaying & Hazarika, 2020, 2022; Yaying et al., 2022; Yaying & Kara, 2021) were investigated.

2. Tetranacci numbers, matrix and sequence spaces

The Tetranacci sequence, initially introduced by Feinberg (1963), is elaborated within this discourse. This sequence, as implied by its nomenclature, is generated through the summation of the preceding four terms. Represented as $(t_n)_{n\in\mathbb{N}}$ for the n^{th} Tetranacci term, the sequence adheres to the recurrence formula

$$t_n = t_{n-1} + t_{n-2} + t_{n-3} + t_{n-4},$$

for $n > 4$, with the sequence initiating from $t_1 = 1$, $t_2 = 1$, $t_3 = 2$, and $t_4 = 4$. Consequently, the first few initial terms of the Tetranacci sequence are given as:

$$1, 1, 2, 4, 8, 15, 29, 56, 108, 208, 401, 773, \cdots.$$

Significant to note from Feinberg's findings are (see (Feinberg, 1963)):

(a) The limit $\lim_{n\to\infty} \frac{t_n}{t_{n+1}}$ converges to 0.51879006 ....

(b) Conversely, $\lim_{n\to\infty} \frac{t_{n+1}}{t_n}$ approaches 1.9275619 ....

Furthermore, the sequence satisfies the identity

$$\sum_{k=1}^{n} t_k = \frac{1}{3}(t_{n+4} - t_{n+2} - 2t_{n+1} - 1)$$

as precisely elucidated in Waddill (1992) and Proposition 2.2 of Soykan's work (Soykan, 2019).

For the purpose of this study, it is stipulated that terms bearing non-positive indices are to be considered as null.

Quite recently, Khan and Meitei (2024) defined the Tetranacci matrix $\mathcal{T}$ and the sequence spaces $\Pi(\mathcal{T})$ as the domain of $\mathcal{T}$ for $\Pi \in \{\ell_p, \ell_\infty, c, c_0\}$ and $1 \leq p < \infty$. Later, authors proved existence theorem with example for infinite systems of differential equations in $\ell_p(\mathcal{T})$ after studying these spaces in terms of some properties such as completeness, isomorphism, inclusion relations, Schauder basis, α-, β- and γ-duals and matrix transformations.

The Tetranacci matrix $\mathcal{T} = (\mathrm{t}_{nk})_{n,k\in\mathbb{N}}$ is defined as

$$\mathrm{t}_{nk} = \begin{cases} \dfrac{3t_k}{t_{n+4} - t_{n+2} - 2t_{n+1} - 1} &, \quad \text{if } 1 \leq k \leq n, \\ 0 &, \quad \text{otherwise.} \end{cases}$$

Equivalently, this can be expressed as

$$\mathcal{T} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & \cdots \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 0 & 0 & \cdots \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{2} & 0 & \cdots \\ \frac{1}{16} & \frac{1}{16} & \frac{1}{8} & \frac{1}{4} & \frac{1}{2} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

The $\mathcal{T}$-transform of any $y = (y_k) \in \omega$ is given by $z = (z_n)$ described by

$$z_n = (\mathcal{T}y)_n = \frac{3}{t_{n+4} - t_{n+2} - 2t_{n+1} - 1} \sum_{k=1}^{n} t_k y_k \ (n \in \mathbb{N}). \tag{2}$$

Lemma 2.1. *$D = (d_{nk})$ is regular iff each of the given conditions*

(i). $\sup_{n\in\mathbb{N}} \sum_k |d_{nk}| < \infty$,

(ii). $\lim_{n\to\infty} \sum_k d_{nk} = 1$,

(iii). $\lim_{n\to\infty} d_{nk} = 0$,

holds true.

The Tetranacci matrix is regular because it satisfies the conditions of the lemma just mentioned. That is, the Tetranacci transform of a convergent sequence converges with the same limit.

The primary objective of this article is to describe to sequence space $\ell_p(\mathcal{T})$ for $0 < p < 1$, explain some properties of the space $\ell_p(\mathcal{T})$ and to give the characterization of a certain class of compact operators acting on $\ell_p(\mathcal{T})$ for $1 \leq p \leq \infty$ by the utilization of Hausdorff measure of non-compactness.

3. Some properties and compactness by Hausdorff measure of non-compactness on $\ell_p(\mathcal{T})$

3.1 The sequence space $\ell_p(\mathcal{T})$ and some properties

In this section, we describe the sequence space $\ell_p(\mathcal{T})$ for $0 < p < 1$, examine some properties for the cases $0 < p < 1$ and $1 \leq p < \infty$, determine the $\alpha -$, $\beta -$ and $\gamma -$duals of the space $\ell_p(\mathcal{T})$ for $0 < p \leq 1$.

We now define the space $\ell_p(\mathcal{T})$ $(0 < p < 1)$ by the following way:

$$\ell_p(\mathcal{T}) \quad = \quad \{y = (y_k) \in \omega: \sum_{n=1}^{\infty} \left| \frac{3}{t_{n+4} - t_{n+2} - 2t_{n+1} - 1} \sum_{k=1}^{n} t_k y_k \right|^p < \infty\}.$$

Theorem 3.1.1. *The space $\ell_p(\mathcal{T})$ is complete p-normed sequence space with the p −norm*

$$\|y\|'_{\ell_p(\mathcal{T})} = \sum_{n=1}^{\infty} \left| \frac{3}{t_{n+4} - t_{n+2} - 2t_{n+1} - 1} \sum_{k=1}^{n} t_k y_k \right|^p$$

for $0 < p < 1$.

Proof. The result follows immediately due to Wilansky (1984) and the fact that ℓ_p is complete p-normed space for $0 < p < 1$ and $\mathcal{T}$ is a triangle.

Theorem 3.1.2. *The space $\ell_p(\mathcal{T})$ is linearly p-norm isomorphic to the space ℓ_p for* $0 < p < 1$.

Proof. Define the function $\mathcal{R}: \ell_p(\mathcal{T}) \to \ell_p$ such that $\mathcal{R}(y) = \mathcal{T}y$ for $0 < p < 1$. The linearity of $\mathcal{R}$ is straightforward.

The injectivity of $\mathcal{R}$ follows from the fact that $\mathcal{R}(y) = 0$ implies $y = 0$.

Let us consider $z = (z_k) \in \ell_p$ and $y = (y_k) \in \omega$ such that

$$y_k = \sum_{i=k-1}^{k} (-1)^{k-i} \frac{t_{i+4} - t_{i+2} - 2t_{i+1} - 1}{3t_k} z_i, \tag{3}$$

with the initial condition $y_1 = z_1$, for every $k \geq 2$. From the relationship

$$\begin{aligned}(\mathcal{T}y)_n &= \frac{3}{t_{n+4} - t_{n+2} - 2t_{n+1} - 1} \sum_{k=1}^{n} t_k y_k \\ &= \frac{3}{t_{n+4} - t_{n+2} - 2t_{n+1} - 1} \sum_{k=1}^{n} t_k \sum_{i=k-1}^{k} (-1)^{k-i} \frac{t_{i+4} - t_{i+2} - 2t_{i+1} - 1}{3t_k} z_i \\ &= z_n,\end{aligned}$$

we see that $\mathcal{R}$ is surjective. Moreover, the equality $\|y\|'_{\ell_p(\mathcal{T})} = \|\mathcal{T}y\|'_{\ell_p}$ shows that $\mathcal{R}$ is norm preserving. This completes the proof.

Theorem 3.1.3. *$\ell_p(\mathcal{T})$ isn't Hilbert space for $1 \leq p < \infty$ and $p \neq 2$.*

Proof. Considering $\tilde{y} = (1,1,-1,0,0,\dots)$ and $\tilde{z} = (1,-3,1,0,0,\dots)$, it follows that $\mathcal{T}\tilde{y} = (1,1,0,0,\dots)$ and $\mathcal{T}\tilde{z} = (1,-1,0,0,\dots)$. Consequently, one derives that

$$\|\tilde{y} + \tilde{z}\|^2_{\ell_p(\mathcal{T})} + \|\tilde{y} - \tilde{z}\|^2_{\ell_p(\mathcal{T})} = 8 \neq 2^{2+\frac{2}{p}} = 2\left(\|\tilde{y}\|^2_{\ell_p(\mathcal{T})} + \|\tilde{z}\|^2_{\ell_p(\mathcal{T})}\right).$$

From this observation, it is evident that the parallelogram law doesn't hold for $\|\cdot\|_{\ell_p(\mathcal{T})}$ when $p \neq 2$, indicating that $\ell_p(\mathcal{T})$ isn't Hilbert space for $1 \leq p < \infty$ and $p \neq 2$.

Theorem 3.1.4. *The inclusion $\ell_p(\mathcal{T}) \subset \ell_s(\mathcal{T})$ is strict for $1 \leq p < s < \infty$.*

Proof. Considering $y = (y_k) \in \ell_p(\mathcal{T})$ such that $\mathcal{T}y \in \ell_p$, and acknowledging that $\ell_p \subset \ell_s$ for $1 \leq p < s < \infty$, it follows that $\mathcal{T}y \in \ell_s$. Therefore, one can assert that $y = (y_k) \in \ell_s(\mathcal{T})$.

The strictness of this inclusion is readily observable from the relation $\tilde{z} = \mathcal{T}\tilde{y} \in \ell_s \setminus \ell_p$.

Given spaces $\Pi, \Theta \subset \omega$, the multiplier set $M(\Pi:\Theta)$ is defined as follows:

$$M(\Pi:\Theta) = \{\lambda = (\lambda_n) \in \omega : \lambda y = (\lambda_n y_n) \in \Theta \text{ for all } (y_n) \in \Pi\}.$$

Then, the α-, β-, and γ-duals of the space Π are given by:

$$\Pi^\alpha = M(\Pi:\ell_1), \Pi^\beta = M(\Pi:cs) \text{ and } \Pi^\gamma = M(\Pi:bs).$$

We now present the following lemmas due to (Gross-Erdmann, 1993; Lascarides & Maddox, 1970), which aid in characterizing certain matrix classes for determining the duals:

Lemma 3.1.5. *For* $0 < p \leq 1$, $D = (d_{nk}) \in (\ell_p : \ell_\infty)$ *iff*

$$\sup_{n,k\in\mathbb{N}} |d_{nk}|^p < \infty. \tag{4}$$

Lemma 3.1.6. *For* $0 < p \leq 1$, $D = (d_{nk}) \in (\ell_p : c)$ *iff the conditions* (4) *and*

$$\exists d_k \in \mathbb{C} \ni \lim_{n\to\infty} d_{nk} = d_k, \text{ for all } \in \mathbb{N}$$

hold.

Lemma 3.1.7. *For* $0 < p \leq 1$, $D = (d_{nk}) \in (\ell_p : \ell_1)$ *iff*

$$\sup_{M\in\mathcal{K}} \sup_{k\in\mathbb{N}} \left| \sum_{n\in\mathcal{K}}^{\infty} d_{nk} \right|^p < \infty. \tag{5}$$

Theorem 3.1.8. *Define the set* ϱ_1 *by*

$$\varrho_1 = \left\{ \lambda = (\lambda_k) \in \omega : \sup_{M\in\mathcal{K}} \sup_{k\in\mathbb{N}} \left| \sum_{n\in\mathcal{K}}^{\infty} g_{nk} \right|^p < \infty \right\},$$

where $G = (g_{nk})$ *is a triangle defined by*

$$g_{nk} = \begin{cases} (-1)^{n-k} \dfrac{t_{k+4} - t_{k+2} - 2t_{k+1} - 1}{3t_n} \lambda_n & , \quad \text{if } n-1 \leq k \leq n, \\ 0 & , \quad \text{otherwise.} \end{cases}$$

Then, $[\ell_p(\mathcal{T})]^\alpha = \varrho_1$ *for* $0 < p \leq 1$.

Proof. By considering the relation (2), we derive the following equality:

$$\begin{aligned} \lambda_n y_n &= \lambda_n \left(\sum_{k=n-1}^{n} (-1)^{n-k} \frac{t_{k+4} - t_{k+2} - 2t_{k+1} - 1}{3t_n} z_k \right) \\ &= \sum_{k=n-1}^{n} \left((-1)^{n-k} \frac{t_{k+4} - t_{k+2} - 2t_{k+1} - 1}{3t_n} \lambda_n \right) z_k = (Gz)_n, \end{aligned} \tag{6}$$

for each $n \in \mathbb{N}$. Consequently, we deduce from relation (6) that $\lambda y = (\lambda_n y_n) \in \ell_1$ whenever $y \in \ell_p(\mathcal{T})$ if and only if $Gz \in \ell_1$ whenever $z \in \ell_p$. This implies that $\lambda \in [\ell_p(\mathcal{T})]^\alpha$ if and only if $G \in (\ell_p : \ell_1)$. Thus, by utilizing Lemma 3.1.7, we conclude that $[\ell_p(\mathcal{T})]^\alpha = \varrho_1$ for $0 < p \leq 1$.

Theorem 3.1.9. *Define the sets* ϱ_2 *and* ϱ_3 *as*

$$\varrho_2 = \left\{\lambda = (\lambda_k) \in \omega: \sup_{n,k\in\mathbb{N}} \left|\left(\frac{\lambda_k}{t_k} - \frac{\lambda_{k+1}}{t_{k+1}}\right)\left(\frac{t_{k+4} - t_{k+2} - 2t_{k+1} - 1}{3}\right)\right|^p < \infty\right\}$$

$$\varrho_3 = \left\{\lambda = (\lambda_k) \in \omega: \left(\frac{t_{k+4} - t_{k+2} - 2t_{k+1} - 1}{3t_k}\lambda_k\right) \in \ell_\infty\right\}.$$

Then, $\left[\ell_p(\mathcal{T})\right]^\beta = \left[\ell_p(\mathcal{T})\right]^\gamma = \varrho_2 \cap \varrho_3$ *for* $0 < p \leq 1$.

Proof. The series $\sum_{k=1}^{\infty} \lambda_k y_k$ converges for the sequences $\lambda = (\lambda_k) \in \left[\ell_p(\mathcal{T})\right]^\beta$ and $y \in \ell_p(\mathcal{T})$ for $0 < p \leq 1$. From the Abel partial sum of the series $\sum_{k=1}^{\infty} \lambda_k y_k$ with (3), we obtain that

$$\begin{aligned}\sum_{k=1}^{n} \lambda_k y_k &= \sum_{k=1}^{n} \lambda_k \left(\sum_{i=k-1}^{k} (-1)^{k-i} \frac{t_{i+4} - t_{i+2} - 2t_{i+1} - 1}{3t_k} z_i\right) \\ &= \sum_{k=1}^{n-1} \left(\frac{\lambda_k}{t_k} - \frac{\lambda_{k+1}}{t_{k+1}}\right)\left(\frac{t_{k+4} - t_{k+2} - 2t_{k+1} - 1}{3}\right) z_k + \frac{t_{n+4} - t_{n+2} - 2t_{n+1} - 1}{3t_n}\lambda_n z_n\end{aligned} \tag{7}$$

for each $n \in \mathbb{N}$. Since $\ell_p(\mathcal{T})$ is linearly isomorphic to ℓ_p, we may pass to the limit as $n \to \infty$ on (7). The convergence of $\sum_{k=1}^{\infty} \lambda_k y_k$ implies that the series $\sum_{k=1}^{\infty} \left(\frac{\lambda_k}{t_k} - \frac{\lambda_{k+1}}{t_{k+1}}\right)\left(\frac{t_{k+4}-t_{k+2}-2t_{k+1}-1}{3}\right) z_k$ is convergent, too and $\frac{t_{n+4}-t_{n+2}-2t_{n+1}-1}{3t_n}\lambda_n \to 0$ as $n \to \infty$. Moreover, since $\ell_p \subset c$, this is possible if $\frac{t_{n+4}-t_{n+2}-2t_{n+1}-1}{3t_n}\lambda_n \in \ell_\infty$. Thus, it is obtained that,

$$\sum_{k=1}^{\infty} \lambda_k y_k = \sum_{k=1}^{\infty} \left(\frac{\lambda_k}{t_k} - \frac{\lambda_{k+1}}{t_{k+1}}\right)\left(\frac{t_{k+4} - t_{k+2} - 2t_{k+1} - 1}{3}\right) z_k = (Oz)_n \tag{8}$$

for all $n \in \mathbb{N}$, where the matrix $O = (o_{nk})_{n,k\in\mathbb{N}}$ is described by $o_{nk} = \left(\frac{\lambda_k}{t_k} - \frac{\lambda_{k+1}}{t_{k+1}}\right)\left(\frac{t_{k+4}-t_{k+2}-2t_{k+1}-1}{3}\right)$. Thus, $O = (o_{nk}) \in (\ell_p: c)$, which implies that $\lambda = (\lambda_k) \in \varrho_2 \cap \varrho_3$, that is $\left[\ell_p(\mathcal{T})\right]^\beta \subset \varrho_2 \cap \varrho_3$.

Conversely, consider that $\lambda = (\lambda_k) \in (\varrho_2 \cap \varrho_3)$ and $z = (z_k) \in \ell_p$. It is obtained the equations (7) and (8). Then the series $\sum_{k=1}^{\infty} \lambda_k y_k$ is convergent for all $y \in \ell_p(\mathcal{T})$, because we have $O = (o_{nk}) \in (\ell_p: c)$. Thus, $\lambda = (\lambda_k) \in \left[\ell_p(\mathcal{T})\right]^\beta$ and consequently $(\varrho_2 \cap \varrho_3) \subset \left[\ell_p(\mathcal{T})\right]^\beta$ for $0 < p \leq 1$.

The γ-dual part can be proven similarly, so we will omit the details.

3.2. Compactness by Hausdorff measure of non-compactness on $\ell_p(\mathcal{T})$ for $1 \leq p \leq \infty$

Consider the unit sphere $\mathcal{N}_\Pi$ of a normed space Π. The acronym $\|.\|_\Pi^{\cdot}$ is described as

$$\|y\|_\Pi^{\cdot} = \sup_{x\in\mathcal{N}_\Pi} \left|\sum_k y_k x_k\right|$$

for any BK-space $\Pi \supset \Upsilon$, $y = (y_k) \in \omega$ and the all finite sequences' space Υ. We assume that the series above exists, and in this case $y \in \Pi^{\beta}$.

Lemma 3.2.1. (Malkowsky & Rakočević, 2000) *The following expressions are provided:*

(i). $c^{\beta} = c_0^{\beta} = \ell_{\infty}^{\beta} = \ell_1$ *and* $\|y\|_{\Pi}^{\cdot} = \|y\|_{\ell_1}$ *for every* $y \in \ell_1$ *and* $\Pi \in \{\ell_{\infty}, c, c_0\}$.

(ii). $\ell_1^{\beta} = \ell_{\infty}$ *and* $\|y\|_{\ell_1}^{\cdot} = \|y\|_{\ell_{\infty}}$ *for every* $y \in \ell_{\infty}$.

(iii). $\ell_p^{\beta} = \ell_q$ *and* $\|y\|_{\ell_p}^{\cdot} = \|y\|_{\ell_q}$ *for every* $y \in \ell_q$.

$\mathfrak{M}(\Pi:\Theta)$ means the collection of all bounded (continuous) linear transformations from Π to Θ.

Lemma 3.2.2. (Malkowsky & Rakočević, 2000) *Suppose that* Π *and* Θ *are BK-spaces. In that case, there is a linear transformation* $\mathcal{V}_D \in \mathfrak{M}(\Pi:\Theta)$ *such that* $\mathcal{V}_D(y) = Dy$ *for all* $y \in \Pi$ *and for any* $D \in (\Pi:\Theta)$.

Lemma 3.2.3. (Malkowsky & Rakočević, 2000) *Let* $\Pi \supset \Upsilon$ *is BK-space. In that case,* $\|\mathcal{V}_D\| = \|D\|_{(\Pi:\Theta)} = \sup_{n\in\mathbb{N}} \|D_n\|_{\Pi}^{\cdot} < \infty$,

for $D \in (\Pi:\Theta)$.

The Hausdorff measure of non-compactness of B, which is any bounded subset of a metric space Π is denoted by $\chi(B)$ with

$$\chi(B) = \inf\{\epsilon > 0: B \subset \cup_{i=1}^{n} O(y_i, m_i), y_i \in \Pi, m_i < \epsilon, n \in \mathbb{N}\},$$

where $O(y_i, m_i)$ represents the open ball with center y_i and radius m_i, where $1 \le i \le n$. Researchers who want to conduct in-depth research on the subject can benefit from source (Malkowsky & Rakočević, 2000).

Theorem 3.2.4. (Malkowsky & Rakočević, 2000) *Let* $B \subset \ell_p$ *is bounded and consider the operator* $\mu_m: \ell_p \longrightarrow \ell_p$ $(m \in \mathbb{N})$ *is defined by* $\mu_m(y) = (y_1, y_2, y_3, \dots, y_m, 0, 0, \dots)$ *for* $y = (y_m) \in \ell_p$, $1 \le p < \infty$. *In this case, for the identity transformation* $\mathcal{I}$ *of* ℓ_p,

$$\chi(B) = \lim_{m\to\infty}\left(\sup_{y\in B} \|(\mathcal{I} - \mu_m)(y)\|_{\ell_p}\right).$$

A linear transformation $\mathcal{V}: \Pi \to \Theta$ is named as compact operator if the sequence $(\mathcal{V}(y))$ possesses a convergent sub-sequence in Θ for all $y = (y_k) \in \Pi \cap \ell_{\infty}$.

The Hausdorff measure of non-compactness $\|\mathcal{V}\|_{\chi}$ of $\mathcal{V}$ is characterized by $\|\mathcal{V}\|_{\chi} = \chi(\mathcal{V}(\mathcal{N}_{\Pi}))$. Thus, a linear transformation $\mathcal{V}$ is compact if and only if $\|\mathcal{V}\|_{\chi} = 0$. For advanced research on the subject, sources (Malkowsky & Rakočević, 2000; Mursaleen & Noman, 2010a, 2010b) can be consulted.

Lemma 3.2.5. (Mursaleen & Noman, 2010a) *Let $\Pi \supset \Upsilon$ be any BK-space. Then:*

(i). *For $D \in (\Pi : c_0)$, $\|\mathcal{V}_D\|_\chi = \limsup_n \|D_n\|_\Pi^{\cdot}$ and $\mathcal{V}_D$ is compact iff $\lim_n \|D_n\|_\Pi^{\cdot} = 0$.*

(ii). *If Π possesses AK property or $\Pi = \ell_\infty$ and $D \in (\Pi : c)$, in this case;*

$$\frac{1}{2}\limsup_n \|D_n - \tau\|_\Pi^{\cdot} \le \|\mathcal{V}_D\|_\chi \le \limsup_n \|D_n - \tau\|_\Pi^{\cdot}$$

and $\mathcal{L}_D$ is compact if

$$\lim_n \|D_n - \tau\|_\Pi^{\cdot} = 0,$$

where $\tau = (\tau_k)$ and $\tau_k = \lim_n d_{nk}$.

(iii). *Let $D \in (\Pi : \ell_\infty)$. Then, $0 \le \|\mathcal{V}_D\|_\chi \le \limsup_n \|D_n\|_\Pi^{\cdot}$ and $\mathcal{V}_D$ is compact if $\lim_n \|D_n\|_\Pi^{\cdot} = 0$.*

(iv). *Let $D \in (\Pi : \ell_1)$. In this case,*

$$\lim_i \left(\sup_{A \in \mathcal{K}_i} \left\| \sum_{n \in A} D_n \right\|_\Pi^{\cdot} \right) \le \|\mathcal{V}_D\|_\chi \le 4 . \lim_i \left(\sup_{A \in \mathcal{K}_i} \left\| \sum_{n \in A} D_n \right\|_\Pi^{\cdot} \right)$$

and $\mathcal{V}_D$ is compact iff $\lim_i \left(\sup_{A \in \mathcal{K}_i} \|\sum_{n \in E} D_n\|_\Pi^{\cdot} \right) = 0$, where $\mathcal{K}$ indicates all finite subsets' class of $\mathbb{N}$ and $\mathcal{K}_i$ indicates subclass of $\mathcal{K}$ consisting of subsets of $\mathbb{N}$ with number of elements m as $m > i$.

In the rest of the paper, we will assume that matrices $\Phi = (\phi_{nk})$ and $D = (d_{nk})$ are connected by the relation

$$\phi_{nk} = \left(\frac{d_{nk}}{t_k} - \frac{d_{n,k+1}}{t_{k+1}} \right) \left(\frac{t_{k+4} - t_{k+2} - 2t_{k+1} - 1}{3} \right) \quad (n, k \in \mathbb{N}). \tag{9}$$

Lemma 3.2.6. *Let $\Theta \subset \omega$ and $1 \le p \le \infty$. If $D \in (\ell_p(\mathcal{T}) : \Theta)$, in this case $\Phi \in (\ell_p : \Theta)$ and $Dy = \Phi z$ is satisfied for all $y \in \ell_p(\mathcal{T})$ with (2).*

Theorem 3.2.7. *Consider that $1 < p < \infty$. In this case:*

(i). *Let $D \in (\ell_p(\mathcal{T}) : c_0)$. Then, $\|\mathcal{V}_D\|_\chi = \limsup_n (\sum_k |\phi_{nk}|^q)^{1/q}$*

and $\mathcal{V}_D$ is compact iff $\lim_n (\sum_k |\phi_{nk}|^q)^{1/q} = 0$.

(ii). *Let $D \in (\ell_p(\mathcal{T}) : c)$. Then,*

$$\frac{1}{2}\limsup_n \left(\sum_k |\phi_{nk} - f_k|^q \right)^{1/q} \le \|\mathcal{V}_D\|_\chi \le \limsup_n \left(\sum_k |\phi_{nk} - f_k|^q \right)^{1/q}$$

and $\mathcal{V}_D$ is compact iff $\lim_n (\sum_k |\phi_{nk} - f_k|^q)^{1/q} = 0$, where $f_k = \lim_n \phi_{nk}$.

(iii). *Let* $D \in (\ell_p(\mathcal{T}):\ell_\infty)$. *In that case,* $0 \le \|\mathcal{V}_D\|_\chi \le \limsup_n (\sum_k |\phi_{nk}|^q)^{1/q}$ *and* $\mathcal{V}_D$ *is compact if* $\lim_n (\sum_k |\phi_{nk}|^q)^{1/q} = 0$.

(iv). *Let* $D \in (\ell_p(\mathcal{T}):\ell_1)$. *Then,* $\lim_i \|D\|^{(i)}_{(\ell_p(\mathcal{T}):\ell_1)} \le \|\mathcal{V}_D\|_\chi \le 4.\lim_i \|D\|^{(i)}_{(\ell_p(\mathcal{T}):\ell_1)}$ *and* $\mathcal{V}_D$ *is compact iff* $\lim_i \|D\|^{(i)}_{(\ell_p(\mathcal{T}):\ell_1)} = 0$, *where* $\|D\|^{(i)}_{(\ell_p(\mathcal{T}):\ell_1)} = \sup_{A\in\mathcal{E}_i} (\sum_k |\sum_{n\in A} \phi_{nk}|^q)^{1/q}$.

Proof. **(i)** Assume that $D \in (\ell_p(\mathcal{T}): c_0)$. Then,

$$\|D_n\|^{\cdot}_{\ell_p(\mathcal{T})} = \|\Phi_n\|^{\cdot}_{\ell_p} = \|\Phi_n\|_{\ell_q} = \left(\sum_k |\phi_{nk}|^q\right)^{1/q}.$$

Consequently, by Lemma 3.2.5/(i), we see the equation

$$\|\mathcal{V}_D\|_\chi = \limsup_n \|D_n\|^{\cdot}_{\ell_p(\mathcal{T})} = \limsup_n \left(\sum_k |\phi_{nk}|^q\right)^{1/q}$$

and $\mathcal{V}_D$ is compact if $\lim_n (\sum_k |\phi_{nk}|^q)^{1/q}$.

(ii) Let $D \in (\ell_p(\mathcal{T}): c)$. Then, $\Phi \in (\ell_p: c)$ by Lemma 3.2.6. By using Lemma 3.2.1/(iii), we see the equation

$$\|\Phi_n - f\|^{\cdot}_{\ell_p} = \|\Phi_n - f\|_{\ell_q} = \left(\sum_k |\phi_{nk} - f_k|^q\right)^{1/q}. \tag{10}$$

By the aid of Lemma 3.2.5/(ii), we observe the inequality

$$\frac{1}{2}\limsup_n \|\Phi_n - f\|^{\cdot}_{\ell_p} \le \|\mathcal{V}_D\|_\chi \le \limsup_n \|\Phi_n - f\|^{\cdot}_{\ell_p}. \tag{11}$$

Then, by considering (10) and (11) together, it is obtained that

$$\frac{1}{2}\limsup_n \left(\sum_k |\phi_{nk} - f_k|^q\right)^{1/q} \le \|\mathcal{V}_D\|_\chi \le \limsup_n \left(\sum_k |\phi_{nk} - f_k|^q\right)^{1/q}.$$

Thus, from Lemma 3.2.5/(ii), $\mathcal{V}_D$ is compact iff

$$\lim_n \left(\sum_k |\phi_{nk} - f_k|^q\right)^{1/q} = 0.$$

(iii) This proof parallels to those of (i) and (ii), by taking into account Lemma 3.2.5/(iii).

(iv) One obtains that

$$\left\|\sum_{n\in A} D_n\right\|^{\cdot}_{\ell_p(\mathcal{T})} = \left\|\sum_{n\in A} \Phi_n\right\|^{\cdot}_{\ell_p} = \left\|\sum_{n\in A} \Phi_n\right\|_{\ell_q} = \left(\sum_k \left|\sum_{n\in A} \phi_{nk}\right|^q\right)^{1/q}.$$

Let $D \in (\ell_p(\mathcal{T}): \ell_1)$, then by Lemma 3.2.6, $\Phi \in (\ell_p: \ell_1)$ holds. By taking into account Lemma 3.2.5/(iv), one concludes that

$$\lim_i \left(\sup_{A\in\mathcal{E}_i} \sum_k \left|\sum_{n\in A} \phi_{nk}\right|^q\right)^{1/q} \leq \|\mathcal{V}_D\|_\chi \leq 4. \lim_i \left(\sup_{A\in\mathcal{E}_i} \sum_k \left|\sum_{n\in A} \phi_{nk}\right|^q\right)^{1/q}$$

and $\mathcal{V}_D$ is compact iff

$$\lim_i \left(\sup_{A\in\mathcal{E}_i} \sum_k \left|\sum_{n\in A} \phi_{nk}\right|^q\right)^{1/q} = 0.$$

Lemma 3.2.8. (Mursaleen & Noman, 2010a) *Let $\Pi \supset \Upsilon$ be BK-space and*

$$\|D\|^{[n]}_{(\Pi:bs)} = \left\|\sum_{r=1}^{n} D_r\right\|^{\cdot}_{\Pi}.$$

The following assertions hold true:

(i). *Let $D \in (\Pi: cs_0)$, then*

$$\|\mathcal{V}_D\|_\chi = \lim_n \sup \|D\|^{[n]}_{(\Pi:bs)}$$

and $\mathcal{V}_D$ is compact iff

$$\lim_n \|D\|^{[n]}_{(\Pi:bs)} = 0.$$

(ii). *Let Π has AK property and $D \in (\Pi: cs)$. In this case,*

$$\frac{1}{2}\lim_n \sup \left\|\sum_{r=1}^{n} D_r - \kappa\right\|^{\cdot}_{\Pi} \leq \|\mathcal{V}_D\|_\chi \leq \lim_n \sup \left\|\sum_{r=1}^{n} D_r - \kappa\right\|^{\cdot}_{\Pi}$$

and $\mathcal{V}_D$ is compact iff

$$\lim_n \|\sum_{r=1}^{n} D_r - \kappa\|^{\cdot}_{\Gamma} = 0,$$

where $\kappa = \kappa_k$ with $\kappa_k = \lim_{n\to\infty} \sum_{r=1}^{n} d_{rk}$ and $k \in \mathbb{N}$.

(iii). *Let $D \in (\Pi: bs)$. In this case,*

$$0 \leq \|\mathcal{V}_D\|_\chi \leq \lim_n \sup \|D\|^{[n]}_{(\Pi:bs)}$$

and $\mathcal{V}_D$ is compact if $\lim_n \|D\|^{[n]}_{(\Pi:bs)} = 0$. and $\mathcal{V}_D$ is compact if

$$\lim_n \left(\sum_k \left|\sum_{r=1}^{n} \phi_{rk}\right|^q\right)^{1/q} = 0.$$

Proof. **(i)** We derive the following equality:

$$\left\|\sum_{r=1}^{n} D_r\right\|_{\ell_p(\mathcal{T})}^{\cdot} = \left\|\sum_{r=1}^{n} \Phi_r\right\|_{\ell_p}^{\cdot} = \left\|\sum_{r=1}^{n} \phi_{rk}\right\|_{\ell_q} = \left(\sum_k \left|\sum_{r=1}^{n} \phi_{rk}\right|^q\right)^{1/q}.$$

Then, from Lemma 3.2.8/(i), we see that $\|\mathcal{V}_D\|_\chi = \limsup_n \left(\sum_k \left|\sum_{r=1}^n \phi_{rk}\right|^q\right)^{1/q}$ and $\mathcal{V}_D$ is compact iff

$$\lim_n \left(\sum_k \left|\sum_{r=1}^{n} \phi_{rk}\right|^q\right)^{1/q} = 0.$$

(ii) We observe that

$$\left\|\sum_{r=1}^{n} \Phi_r - \tilde{f}\right\|_{\ell_p}^{\cdot} = \left\|\sum_{r=1}^{n} \Phi_r - \tilde{f}\right\|_{\ell_q} = \left(\sum_k \left|\sum_{r=1}^{n} \phi_{rk} - \tilde{f}\right|^q\right)^{1/q}. \tag{12}$$

Let $D \in (\ell_p(\mathcal{T}): cs)$. By utilizing Lemma 3.2.6, one obtains $\Phi \in (\ell_p: cs)$. Thus, from Lemma 3.2.8/(ii), one deduces that

$$\frac{1}{2}\limsup_n \left\|\sum_{r=1}^{n} \Phi_r - \tilde{f}\right\|_{\ell_p}^{\cdot} \le \|\mathcal{V}_D\|_\chi \le \limsup_n \left\|\sum_{r=1}^{n} \Phi_r - \tilde{f}\right\|_{\ell_p}^{\cdot},$$

which on using (12) gives us

$$\frac{1}{2}\limsup_n \left(\sum_k \left|\sum_{r=1}^{n} \phi_{rk} - \tilde{f}\right|^q\right)^{1/q} \le \|\mathcal{V}_D\|_\chi \le \limsup_n \left(\sum_k \left|\sum_{r=1}^{n} \phi_{rk} - \tilde{f}\right|^q\right)^{1/q}$$

and also, $\mathcal{V}_D$ is compact iff

$$\lim_n \left(\sum_k \left|\sum_{r=1}^{n} \phi_{rk} - \tilde{f}_k\right|^q\right)^{1/q} = 0.$$

(iii) The proof mirrors the approach of the first part and by utilizing Lemma 3.2.8/(iii).

Theorem 3.2.9.

(i). *Let $D \in (\ell_\infty(\mathcal{T}): c_0)$. Then, $\|\mathcal{V}_D\|_\chi = \limsup_n \sum_k |\phi_{nk}|$ and $\mathcal{V}_D$ is compact if $\lim_n \sum_k |\phi_{nk}| = 0$.*

(ii). *Let $D \in (\ell_\infty(\mathcal{T}): c)$. In this case,*

$$\frac{1}{2}\limsup_n \left(\sum_k |\phi_{nk} - f_k|\right) \le \|\mathcal{V}_D\|_\chi \le \limsup_n \left(\sum_k |\phi_{nk} - f_k|\right)$$

and $\mathcal{V}_D$ is compact iff $\lim_n \left(\sum_k |\phi_{nk} - f_k|\right) = 0$.

(iii). *Let $D \in (\ell_\infty(\mathcal{T}):\ell_\infty)$. In this case,*

$$0 \leq \|\mathcal{V}_D\|_\chi \leq \limsup_n \sum_k |\phi_{nk}|$$

and $\mathcal{V}_D$ is compact if $\lim_n \sum_k |\phi_{nk}| = 0$.

(iv). *Let $D \in (\ell_\infty(\mathcal{T}):\ell_1)$. Then,*

$$\lim_i \|D\|^{(i)}_{(\ell_\infty(\mathcal{T}):\ell_1)} \leq \|\mathcal{V}_D\|_\chi \leq 4. \lim_i \|D\|^{(i)}_{(\ell_\infty(\mathcal{T}):\ell_1)},$$

and $\mathcal{V}_D$ is compact iff $\lim_i \|D\|^{(i)}_{(\ell_\infty(\mathcal{T}):\ell_1)} = 0$, *where*

$$\|D\|^{(i)}_{(\ell_\infty(\mathcal{T}):\ell_1)} = \sup_{A \in \mathcal{E}_i} \left(\sum_k \left| \sum_{n \in A} \phi_{nk} \right| \right).$$

.

Proof. This parallels to the approach employed in Theorem 3.2.7 and therefore it is omitted for brevity.

Theorem 3.2.10.

(i). *Let $D \in (\ell_1(\mathcal{T}):c_0)$. Then,* $\|\mathcal{V}_D\|_\chi = \limsup_n (\sup_k |\phi_{nk}|)$ *and $\mathcal{V}_D$ is compact iff* $\lim_n (\sup_k |\phi_{nk}|) = 0$.

(ii). *Let $D \in (\ell_1(\mathcal{T}):c)$. Then,*

$$\frac{1}{2} \limsup_n \left(\sup_k |\phi_{nk} - f_k| \right) \leq \|\mathcal{V}_D\|_\chi \leq \limsup_n \left(\sup_k |\phi_{nk} - f_k| \right)$$

and $\mathcal{V}_D$ is compact iff $\lim_n \left(\sup_k |\phi_{nk} - f_k| \right) = 0$.

(iii). *Let $D \in (\ell_1(\mathcal{T}):\ell_\infty)$. Then,* $0 \leq \|\mathcal{V}_D\|_\chi \leq \limsup_n \left(\sup_k |\phi_{nk}| \right)$

and $\mathcal{V}_D$ is compact if $\lim_n \left(\sup_k |\phi_{nk}| \right) = 0$.

Proof. The proof parallels to the approach employed in Theorem 3.2.7 and therefore it is omitted for brevity.

4. Conclusion

Measures of non-compactness have a wide scope in functional analysis. These are also applied in metric fixed point theory, in the operator equations' theory in Banach spaces, and in the study of varied differential and integral equations. In particular, the characterization of compact operators on BK-spaces, which is the basis of our work, makes use of the Hausdorff measure of non-compactness.

As outlined above, in this study, some properties of $\ell_p(\mathcal{T})$ for the cases $0 < p < 1$ and $1 \leq p < \infty$ are examined and the compactness criteria of the matrix operators on the sequence spaces $\ell_p(\mathcal{T})$ for $1 \leq p \leq \infty$ defined by Khan and Meitei (2024) are characterized with by using Hausdorff measure of non-compactness.

In our subsequent studies, we will focus on the main topic of determining the compactness criteria of matrix operators defined in sequence spaces by the aid of Hausdorff measure of non-compactness.

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